

Light Quarks (u, d, s)

OMITTED FROM SUMMARY TABLE

u -QUARK MASS

The u -, d -, and s -quark masses are estimates of so-called “current-quark masses,” in a mass-independent subtraction scheme such as $\overline{\text{MS}}$. The ratios m_u/m_d and m_s/m_d are extracted from pion and kaon masses using chiral symmetry. The estimates of d and u masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the u quark could be essentially massless. The s -quark mass is estimated from SU(3) splittings in hadron masses.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of “Our Evaluation” were determined in part via Figures 1 and 2.

$\overline{\text{MS}}$ MASS (MeV)	DOCUMENT ID	TECN
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2.16 $\begin{matrix} +0.49 \\ -0.26 \end{matrix}$ OUR EVALUATION	See the ideogram below.
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2.6 ± 0.4	1 DOMINGUEZ 19 THEO
2.130 ± 0.041	2 BAZAVOV 18 LATT
2.27 $\pm 0.06 \pm 0.06$	3 FODOR 16 LATT
2.36 ± 0.24	4 CARRASCO 14 LATT
2.57 $\pm 0.26 \pm 0.07$	5 AOKI 12 LATT
2.24 $\pm 0.10 \pm 0.34$	6 BLUM 10 LATT
2.01 ± 0.14	7 MCNEILE 10 LATT

• • • We do not use the following data for averages, fits, limits, etc. • • •

2.15 $\pm 0.03 \pm 0.10$	8 DURR 11 LATT
1.9 ± 0.2	9 BAZAVOV 10 LATT
2.01 ± 0.14	7 DAVIES 10 LATT
2.9 ± 0.2	10 DOMINGUEZ 09 THEO
2.9 ± 0.8	11 DEANDREA 08 THEO
3.02 ± 0.33	12 BLUM 07 LATT
2.7 ± 0.4	13 JAMIN 06 THEO
1.9 ± 0.2	14 MASON 06 LATT
2.8 ± 0.2	15 NARISON 06 THEO
1.7 ± 0.3	16 AUBIN 04A LATT

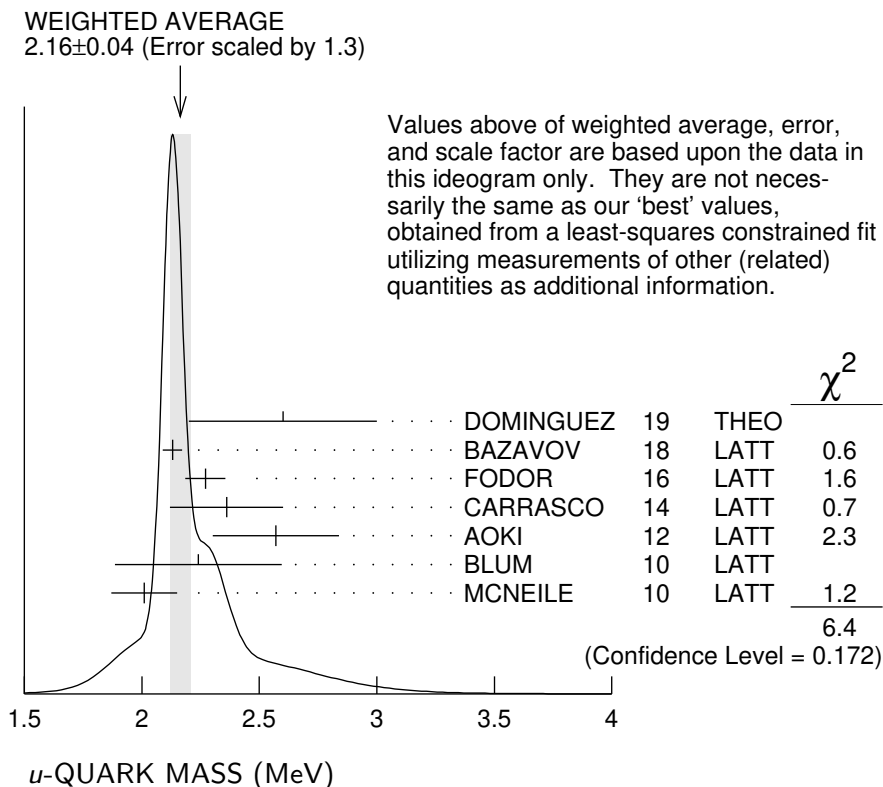
¹ DOMINGUEZ 19 determine the quark mass from a QCD finite energy sum rule for the divergence of the axial current.

² BAZAVOV 18 determine the quark masses using a lattice computation with staggered fermions and four active quark flavors.

³ FODOR 16 is a lattice simulation with $N_f = 2 + 1$ dynamical flavors and includes partially quenched QED effects.

⁴ CARRASCO 14 is a lattice QCD computation of light quark masses using $2 + 1 + 1$ dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

⁵ AOKI 12 is a lattice computation using $1 + 1 + 1$ dynamical quark flavors.



- ⁶ BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use 2+1 dynamical quark flavors.
- ⁷ DAVIES 10 and MCNEILE 10 determine $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass m_u is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratios, m_s/\overline{m} and m_u/m_d .
- ⁸ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed. The individual m_u , m_d values are obtained using the lattice determination of the average mass m_{ud} and of the ratio m_s/m_{ud} and the value of $Q = (m_s^2 - m_{ud}^2) / (m_d^2 - m_u^2)$ as determined from $\eta \rightarrow 3\pi$ decays.
- ⁹ BAZAVOV 10 is a lattice computation using 2+1 dynamical quark flavors.
- ¹⁰ DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order α_s^4 .
- ¹¹ DEANDREA 08 determine $m_u - m_d$ from $\eta \rightarrow 3\pi^0$, and combine with the PDG 06 lattice average value of $m_u + m_d = 7.6 \pm 1.6$ to determine m_u and m_d .
- ¹² BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- ¹³ JAMIN 06 determine $m_u(2 \text{ GeV})$ by combining the value of m_s obtained from the spectral function for the scalar $K\pi$ form factor with other determinations of the quark mass ratios.
- ¹⁴ MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order. The quark masses

m_u and m_d were determined from their $(m_u+m_d)/2$ measurement and AUBIN 04A m_u/m_d value.

¹⁵ NARISON 06 uses sum rules for $e^+e^- \rightarrow$ hadrons to order α_s^3 to determine m_s combined with other determinations of the quark mass ratios.

¹⁶ AUBIN 04A employ a partially quenched lattice calculation of the pseudoscalar meson masses.

d-QUARK MASS

See the comment for the u quark above.

We have normalized the \overline{MS} masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

\overline{MS} MASS (MeV)	DOCUMENT ID	TECN
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4.67 ± 0.48 -0.17	OUR EVALUATION	See the ideogram below.
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5.3 ± 0.4	1 DOMINGUEZ 19	THEO
4.675 ± 0.056	2 BAZAVOV 18	LATT
4.67 $\pm 0.06 \pm 0.06$	3 FODOR 16	LATT
5.03 ± 0.26	4 CARRASCO 14	LATT
3.68 $\pm 0.29 \pm 0.10$	5 AOKI 12	LATT
4.65 $\pm 0.15 \pm 0.32$	6 BLUM 10	LATT
4.77 ± 0.15	7 MCNEILE 10	LATT

• • • We do not use the following data for averages, fits, limits, etc. • • •

4.79 $\pm 0.07 \pm 0.12$	8 DURR 11	LATT
4.6 ± 0.3	9 BAZAVOV 10	LATT
4.79 ± 0.16	7 DAVIES 10	LATT
5.3 ± 0.4	10 DOMINGUEZ 09	THEO
4.7 ± 0.8	11 DEANDREA 08	THEO
5.49 ± 0.39	12 BLUM 07	LATT
4.8 ± 0.5	13 JAMIN 06	THEO
4.4 ± 0.3	14 MASON 06	LATT
5.1 ± 0.4	15 NARISON 06	THEO
3.9 ± 0.5	16 AUBIN 04A	LATT

¹ DOMINGUEZ 19 determine the quark mass from a QCD finite energy sum rule for the divergence of the axial current.

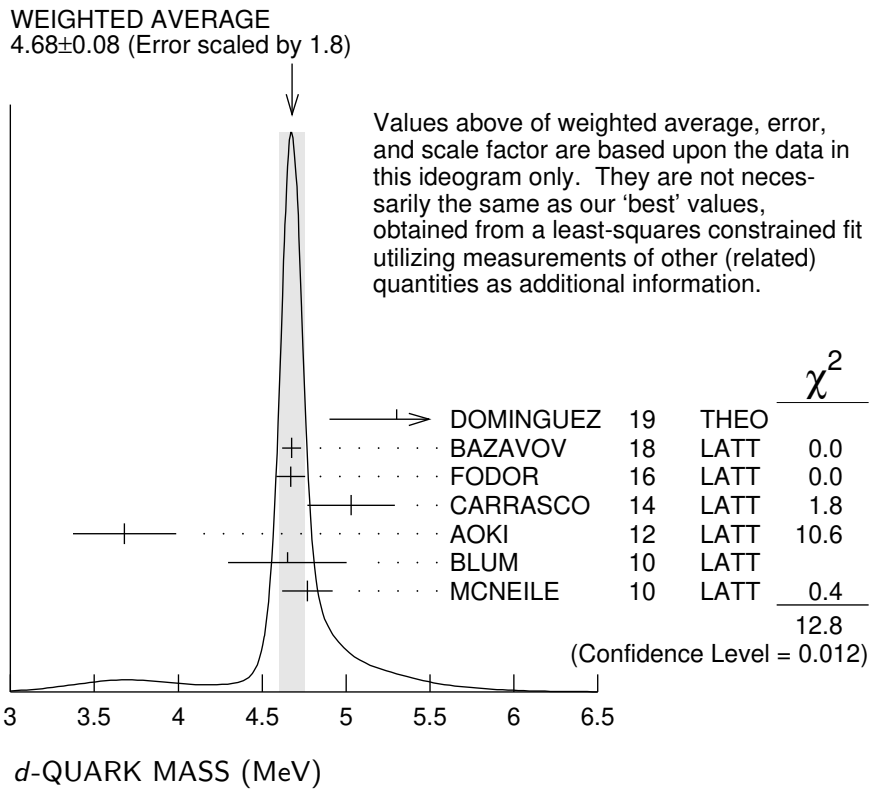
² BAZAVOV 18 determine the quark masses using a lattice computation with staggered fermions and four active quark flavors.

³ FODOR 16 is a lattice simulation with $N_f = 2 + 1$ dynamical flavors and includes partially quenched QED effects.

⁴ CARRASCO 14 is a lattice QCD computation of light quark masses using $2 + 1 + 1$ dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

⁵ AOKI 12 is a lattice computation using $1 + 1 + 1$ dynamical quark flavors.

⁶ BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use $2+1$ dynamical quark flavors.



⁷ DAVIES 10 and MCNEILE 10 determine $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass m_d is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratios, m_s/\overline{m} and m_u/m_d .

⁸ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed. The individual m_u , m_d values are obtained using the lattice determination of the average mass m_{ud} and of the ratio m_s/m_{ud} and the value of $Q = (m_s^2 - m_{ud}^2) / (m_d^2 - m_u^2)$ as determined from $\eta \rightarrow 3\pi$ decays.

⁹ BAZAVOV 10 is a lattice computation using 2+1 dynamical quark flavors.

¹⁰ DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order α_s^4 .

¹¹ DEANDREA 08 determine $m_u - m_d$ from $\eta \rightarrow 3\pi^0$, and combine with the PDG 06 lattice average value of $m_u + m_d = 7.6 \pm 1.6$ to determine m_u and m_d .

¹² BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.

¹³ JAMIN 06 determine $m_d(2 \text{ GeV})$ by combining the value of m_s obtained from the spectral function for the scalar $K\pi$ form factor with other determinations of the quark mass ratios.

¹⁴ MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order. The quark masses m_u and m_d were determined from their $(m_u + m_d)/2$ measurement and AUBIN 04A m_u/m_d value.

¹⁵ NARISON 06 uses sum rules for $e^+ e^- \rightarrow$ hadrons to order α_s^3 to determine m_s combined with other determinations of the quark mass ratios.

¹⁶ AUBIN 04A perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with continuum estimate of electromagnetic effects in the kaon masses, and one-loop perturbative renormalization constant.

$$\bar{m} = (m_u + m_d)/2$$

See the comments for the u quark above.

We have normalized the \overline{MS} masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

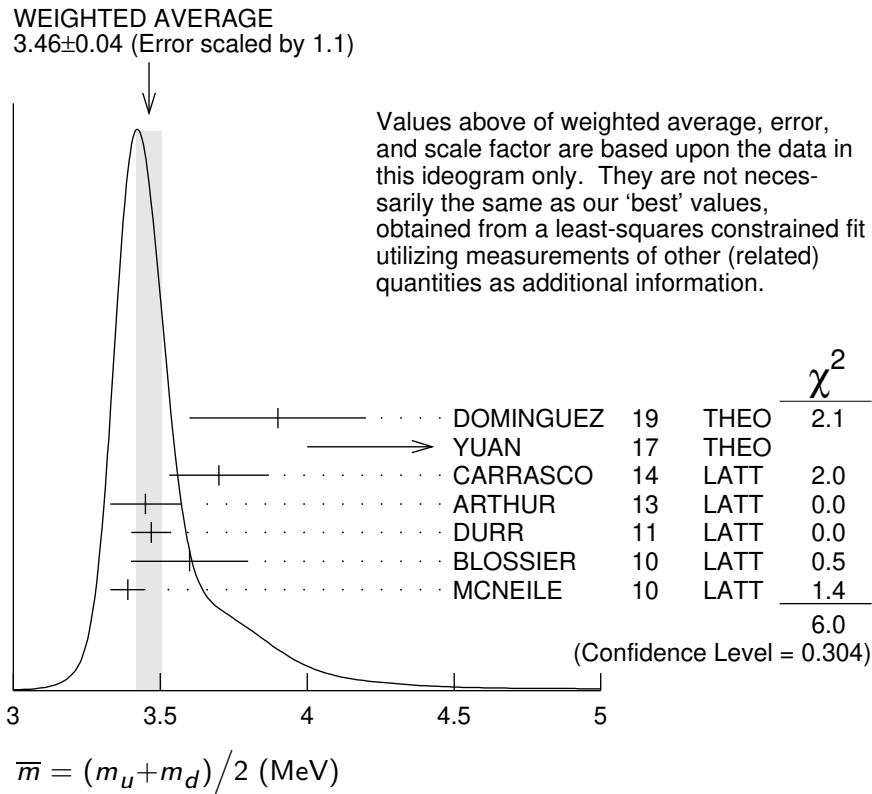
\overline{MS} MASS (MeV)	DOCUMENT ID	TECN
3.45 $\begin{smallmatrix} +0.55 \\ -0.15 \end{smallmatrix}$ OUR EVALUATION	See the ideogram below.	
3.9 ± 0.3	¹ DOMINGUEZ 19	THEO
4.7 $\begin{smallmatrix} +0.8 \\ -0.7 \end{smallmatrix}$	² YUAN	17 THEO
3.70 ± 0.17	³ CARRASCO	14 LATT
3.45 ± 0.12	⁴ ARTHUR	13 LATT
3.469 $\pm 0.047 \pm 0.048$	⁵ DURR	11 LATT
3.6 ± 0.2	⁶ BLOSSIER	10 LATT
3.39 ± 0.06	⁷ MCNEILE	10 LATT
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●		
3.59 ± 0.21	⁸ AOKI	11A LATT
3.40 ± 0.07	⁷ DAVIES	10 LATT
4.1 ± 0.2	⁹ DOMINGUEZ	09 THEO
3.72 ± 0.41	¹⁰ ALLTON	08 LATT
3.85 $\pm 0.12 \pm 0.4$	¹¹ BLOSSIER	08 LATT
\geq 4.85 ± 0.20	¹² DOMINGUEZ...08B	THEO
3.55 $\begin{smallmatrix} +0.65 \\ -0.28 \end{smallmatrix}$	¹³ ISHIKAWA	08 LATT
4.026 ± 0.048	¹⁴ NAKAMURA	08 LATT
4.25 ± 0.35	¹⁵ BLUM	07 LATT
4.08 $\pm 0.25 \pm 0.42$	¹⁶ GOCKELER	06 LATT
4.7 $\pm 0.2 \pm 0.3$	¹⁷ GOCKELER	06A LATT
3.2 ± 0.3	¹⁸ MASON	06 LATT
3.95 ± 0.3	¹⁹ NARISON	06 THEO
2.8 ± 0.3	²⁰ AUBIN	04 LATT
4.29 $\pm 0.14 \pm 0.65$	²¹ AOKI	03 LATT
3.223 ± 0.3	²² AOKI	03B LATT
4.4 $\pm 0.1 \pm 0.4$	²³ BECIREVIC	03 LATT
4.1 $\pm 0.3 \pm 1.0$	²⁴ CHIU	03 LATT

¹ DOMINGUEZ 19 determine the quark mass from a QCD finite energy sum rule for the divergence of the axial current.

² YUAN 17 determine \bar{m} using QCD sum rules in the isospin $I=0$ scalar channel. At the end of the "Numerical Results" section of YUAN 17 the authors discuss the significance of their larger value of the light quark mass compared to previous determinations.

- ³ CARRASCO 14 is a lattice QCD computation of light quark masses using $2 + 1 + 1$ dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.
- ⁴ ARTHUR 13 is a lattice computation using $2+1$ dynamical domain wall fermions. Masses at $\mu = 3$ GeV have been converted to $\mu = 2$ GeV using conversion factors given in their paper.
- ⁵ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.
- ⁶ BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using $N_f=2$ dynamical twisted-mass Wilson fermions.
- ⁷ DAVIES 10 and MCNEILE 10 determine $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass \overline{m} is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratio, m_s/\overline{m} .
- ⁸ AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using $N_f = 2 + 1$ dynamical flavors of domain wall fermions.
- ⁹ DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order α_s^4 .
- ¹⁰ ALLTON 08 use a lattice computation of the π , K , and Ω masses with $2+1$ dynamical flavors of domain wall quarks, and non-perturbative renormalization.
- ¹¹ BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.
- ¹² DOMINGUEZ-CLARIMON 08B obtain an inequality from sum rules for the scalar two-point correlator.
- ¹³ ISHIKAWA 08 use a lattice computation of the light meson spectrum with $2+1$ dynamical flavors of $\mathcal{O}(a)$ improved Wilson quarks, and one-loop perturbative renormalization.
- ¹⁴ NAKAMURA 08 do a lattice computation using quenched domain wall fermions and non-perturbative renormalization.
- ¹⁵ BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- ¹⁶ GOCKELER 06 use an unquenched lattice computation of the axial Ward Identity with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization, to obtain $\overline{m}(2 \text{ GeV}) = 4.08 \pm 0.25 \pm 0.19 \pm 0.23 \text{ MeV}$, where the first error is statistical, the second and third are systematic due to the fit range and force scale uncertainties, respectively. We have combined the systematic errors linearly.
- ¹⁷ GOCKELER 06A use an unquenched lattice computation of the pseudoscalar meson masses with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization.
- ¹⁸ MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order.
- ¹⁹ NARISON 06 uses sum rules for $e^+ e^- \rightarrow$ hadrons to order α_s^3 to determine m_s combined with other determinations of the quark mass ratios.
- ²⁰ AUBIN 04 perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with one-loop perturbative renormalization constant.
- ²¹ AOKI 03 uses quenched lattice simulation of the meson and baryon masses with degenerate light quarks. The extrapolations are done using quenched chiral perturbation theory.
- ²² The errors given in AOKI 03B were $\begin{matrix} +0.046 \\ -0.069 \end{matrix}$. We changed them to ± 0.3 for calculating the overall best values. AOKI 03B uses lattice simulation of the meson and baryon masses with two dynamical light quarks. Simulations are performed using the $\mathcal{O}(a)$ improved Wilson action.
- ²³ BECIREVIC 03 perform quenched lattice computation using the vector and axial Ward identities. Uses $\mathcal{O}(a)$ improved Wilson action and nonperturbative renormalization.

²⁴ CHIU 03 determines quark masses from the pion and kaon masses using a lattice simulation with a chiral fermion action in quenched approximation.



m_u/m_d MASS RATIO

VALUE	DOCUMENT ID	TECN	COMMENT
0.47 $\begin{smallmatrix} +0.06 \\ -0.07 \end{smallmatrix}$	OUR EVALUATION		See the ideogram below.
0.485 ± 0.011 ± 0.016	¹ FODOR	16	LATT
0.4482 $\begin{smallmatrix} +0.0173 \\ -0.0206 \end{smallmatrix}$	² BASAK	15	LATT
0.470 ± 0.056	³ CARRASCO	14	LATT
0.698 ± 0.051	⁴ AOKI	12	LATT
0.42 ± 0.01 ± 0.04	⁵ BAZAVOV	10	LATT
0.4818 ± 0.0096 ± 0.0860	⁶ BLUM	10	LATT
• • • We do not use the following data for averages, fits, limits, etc. • • •			
0.550 ± 0.031	⁷ BLUM	07	LATT
0.43 ± 0.08	⁸ AUBIN	04A	LATT
0.410 ± 0.036	⁹ NELSON	03	LATT
0.553 ± 0.043	¹⁰ LEUTWYLER	96	THEO Compilation

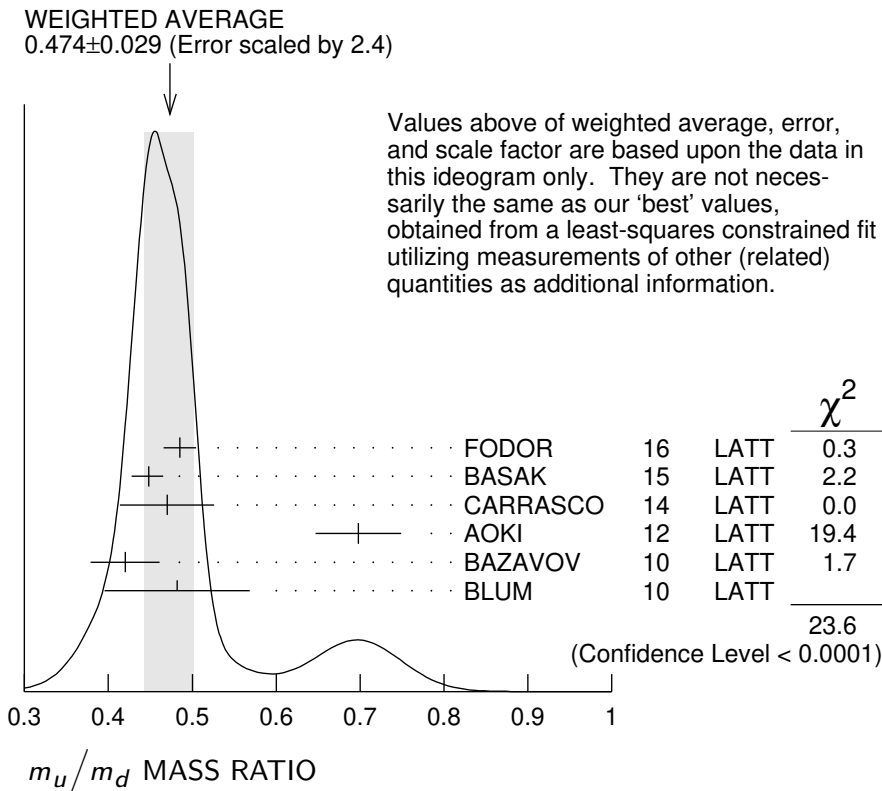
¹ FODOR 16 is a lattice simulation with $N_f = 2 + 1$ dynamical flavors and includes partially quenched QED effects.

² BASAK 15 is a lattice computation using 2+1 dynamical quark flavors.

³ CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are

obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

- ⁴ AOKI 12 is a lattice computation using 1 + 1 + 1 dynamical quark flavors.
- ⁵ BAZAVOV 10 is a lattice computation using 2+1 dynamical quark flavors.
- ⁶ BLUM 10 is a lattice computation using 2+1 dynamical quark flavors.
- ⁷ BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- ⁸ AUBIN 04A perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with continuum estimate of electromagnetic effects in the kaon masses.
- ⁹ NELSON 03 computes coefficients in the order p^4 chiral Lagrangian using a lattice calculation with three dynamical flavors. The ratio m_u/m_d is obtained by combining this with the chiral perturbation theory computation of the meson masses to order p^4 .
- ¹⁰ LEUTWYLER 96 uses a combined fit to $\eta \rightarrow 3\pi$ and $\psi' \rightarrow J/\psi (\pi, \eta)$ decay rates, and the electromagnetic mass differences of the π and K .



s-QUARK MASS

See the comment for the u quark above.

We have normalized the \overline{MS} masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35.

\overline{MS} MASS (MeV) DOCUMENT ID TECN

93 $\pm \frac{11}{5}$ **OUR EVALUATION** See the ideogram below.

92.47 ± 0.69	1	BAZAVOV	18	LATT
93.85 ± 0.75	2	LYTLE	18	LATT

87.6 ± 6.0	3	ANANTHANA..16	THEO
99.6 ± 4.3	4	CARRASCO 14	LATT
94.4 ± 2.3	5	ARTHUR 13	LATT
94 ± 9	6	BODENSTEIN 13	THEO
102 ± 3 ± 1	7	FRITZSCH 12	LATT
95.5 ± 1.1 ± 1.5	8	DURR 11	LATT
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●			
93.6 ± 0.8	9	CHAKRABOR..15	LATT
96.2 ± 2.7	10	AOKI 11A	LATT
95 ± 6	11	BLOSSIER 10	LATT
97.6 ± 2.9 ± 5.5	12	BLUM 10	LATT
92.4 ± 1.5	13	DAVIES 10	LATT
92.2 ± 1.3	13	MCNEILE 10	LATT
107.3 ± 11.7	14	ALLTON 08	LATT
105 ± 3 ± 9	15	BLOSSIER 08	LATT
102 ± 8	16	DOMINGUEZ 08A	THEO
90.1 +17.2 - 6.1	17	ISHIKAWA 08	LATT
105.6 ± 1.2	18	NAKAMURA 08	LATT
119.5 ± 9.3	19	BLUM 07	LATT
105 ± 6 ± 7	20	CHETYRKIN 06	THEO
111 ± 6 ± 10	21	GOCKELER 06	LATT
119 ± 5 ± 8	22	GOCKELER 06A	LATT
92 ± 9	23	JAMIN 06	THEO
87 ± 6	24	MASON 06	LATT
104 ± 15	25	NARISON 06	THEO
≥ 71 ± 4, ≤ 151 ± 14	26	NARISON 06	THEO
96 + 5 +16 - 3 -18	27	BAIKOV 05	THEO
81 ± 22	28	GAMIZ 05	THEO
125 ± 28	29	GORBUNOV 05	THEO
93 ± 32	30	NARISON 05	THEO
76 ± 8	31	AUBIN 04	LATT
116 ± 6 ± 0.65	32	AOKI 03	LATT
84.5 +12 - 1.7	33	AOKI 03B	LATT
106 ± 2 ± 8	34	BEQUIREVIC 03	LATT
92 ± 9 ± 16	35	CHIU 03	LATT
117 ± 17	36	GAMIZ 03	THEO
103 ± 17	37	GAMIZ 03	THEO

¹BAZAVOV 18 determine the quark masses using a lattice computation with staggered fermions and four active quark flavors.

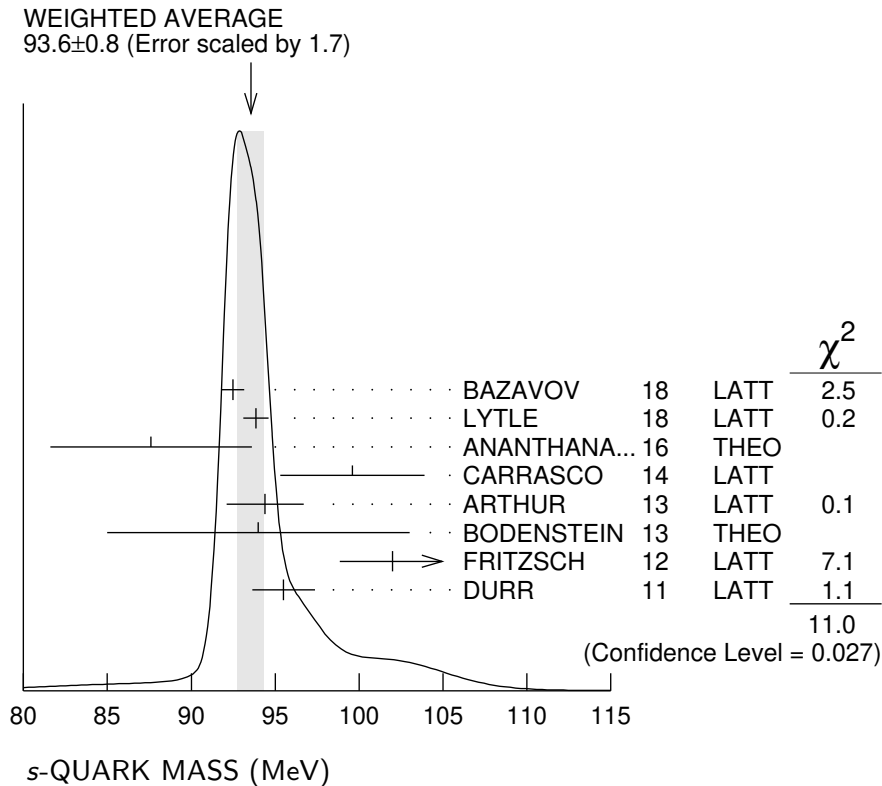
²LYTLE 18 combined with CHAKRABORTY 2015 determine $\overline{m}_s(3 \text{ GeV}) = 84.78 \pm 0.65$ MeV from a lattice simulation with $n_f = 2+1+1$ flavors. They also determine the quoted value $\overline{m}_s(2 \text{ GeV})$ for $n_f = 4$ dynamical flavors.

³ANANTHANARAYAN 16 determine $\overline{m}_s(2 \text{ GeV}) = 106.70 \pm 9.36$ MeV and 74.47 ± 7.77 MeV from fits to ALEPH and OPAL τ decay data, respectively. We have used the weighted average of the two.

⁴CARRASCO 14 is a lattice QCD computation of light quark masses using $2 + 1 + 1$ dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

- 5 ARTHUR 13 is a lattice computation using 2+1 dynamical domain wall fermions. Masses at $\mu = 3$ GeV have been converted to $\mu = 2$ GeV using conversion factors given in their paper.
- 6 BODENSTEIN 13 determines m_S from QCD finite energy sum rules, and the perturbative computation of the pseudoscalar correlator to five-loop order.
- 7 FRITZSCH 12 determine m_S using a lattice computation with $N_f = 2$ dynamical flavors.
- 8 DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.
- 9 CHAKRABORTY 15 is a lattice QCD computation that determines m_C and m_C/m_S using pseudoscalar mesons masses tuned on gluon field configurations with 2+1+1 dynamical flavors of HISQ quarks with u/d masses down to the physical value.
- 10 AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using $N_f = 2 + 1$ dynamical flavors of domain wall fermions.
- 11 BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using $N_f=2$ dynamical twisted-mass Wilson fermions.
- 12 BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use 2+1 dynamical quark flavors.
- 13 DAVIES 10 and MCNEILE 10 determine $\overline{m}_C(\mu)/\overline{m}_S(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass m_S is obtained from this using the value of m_C from ALLISON 08 or MCNEILE 10.
- 14 ALLTON 08 use a lattice computation of the π , K , and Ω masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.
- 15 BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.
- 16 DOMINGUEZ 08A make determination from QCD finite energy sum rules for the pseudoscalar two-point function computed to order α_s^4 .
- 17 ISHIKAWA 08 use a lattice computation of the light meson spectrum with 2+1 dynamical flavors of $\mathcal{O}(a)$ improved Wilson quarks, and one-loop perturbative renormalization.
- 18 NAKAMURA 08 do a lattice computation using quenched domain wall fermions and non-perturbative renormalization.
- 19 BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- 20 CHETYRKIN 06 use QCD sum rules in the pseudoscalar channel to order α_s^4 .
- 21 GOCKELER 06 use an unquenched lattice computation of the axial Ward Identity with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization, to obtain $\overline{m}_S(2 \text{ GeV}) = 111 \pm 6 \pm 4 \pm 6 \text{ MeV}$, where the first error is statistical, the second and third are systematic due to the fit range and force scale uncertainties, respectively. We have combined the systematic errors linearly.
- 22 GOCKELER 06A use an unquenched lattice computation of the pseudoscalar meson masses with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization.
- 23 JAMIN 06 determine $\overline{m}_S(2 \text{ GeV})$ from the spectral function for the scalar $K\pi$ form factor.
- 24 MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order.
- 25 NARISON 06 uses sum rules for $e^+ e^- \rightarrow$ hadrons to order α_s^3 .
- 26 NARISON 06 obtains the quoted range from positivity of the spectral functions.
- 27 BAIKOV 05 determines $\overline{m}_S(M_\tau) = 100^{+5+17}_{-3-19}$ from sum rules using the strange spectral function in τ decay. The computations were done to order α_s^3 , with an estimate of the α_s^4 terms. We have converted the result to $\mu = 2$ GeV.

- 28 GAMIZ 05 determines $\overline{m}_s(2 \text{ GeV})$ from sum rules using the strange spectral function in τ decay. The computations were done to order α_s^2 , with an estimate of the α_s^3 terms.
- 29 GORBUNOV 05 use hadronic tau decays to N3LO, including power corrections.
- 30 NARISON 05 determines $\overline{m}_s(2 \text{ GeV})$ from sum rules using the strange spectral function in τ decay. The computations were done to order α_s^3 .
- 31 AUBIN 04 perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with one-loop perturbative renormalization constant.
- 32 AOKI 03 uses quenched lattice simulation of the meson and baryon masses with degenerate light quarks. The extrapolations are done using quenched chiral perturbation theory. Determines $m_s=113.8 \pm 2.3^{+5.8}_{-2.9}$ using K mass as input and $m_s=142.3 \pm 5.8^{+22}_0$ using ϕ mass as input. We have performed a weighted average of these values.
- 33 AOKI 03B uses lattice simulation of the meson and baryon masses with two dynamical light quarks. Simulations are performed using the $\mathcal{O}(a)$ improved Wilson action.
- 34 BECIREVIC 03 perform quenched lattice computation using the vector and axial Ward identities. Uses $\mathcal{O}(a)$ improved Wilson action and nonperturbative renormalization. They also quote $\overline{m}/m_s=24.3 \pm 0.2 \pm 0.6$.
- 35 CHIU 03 determines quark masses from the pion and kaon masses using a lattice simulation with a chiral fermion action in quenched approximation.
- 36 GAMIZ 03 determines m_s from SU(3) breaking in the τ hadronic width. The value of V_{us} is chosen to satisfy CKM unitarity.
- 37 GAMIZ 03 determines m_s from SU(3) breaking in the τ hadronic width. The value of V_{us} is taken from the PDG.



OTHER LIGHT QUARK MASS RATIOS

 m_s/m_d MASS RATIO

<u>VALUE</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>COMMENT</u>
17–22 OUR EVALUATION			
20.0	1 GAO	97	THEO
18.9 ± 0.8	2 LEUTWYLER	96	THEO Compilation
21	3 DONOGHUE	92	THEO
18	4 GERARD	90	THEO
18 to 23	5 LEUTWYLER	90B	THEO

¹ GAO 97 uses electromagnetic mass splittings of light mesons.

² LEUTWYLER 96 uses a combined fit to $\eta \rightarrow 3\pi$ and $\psi' \rightarrow J/\psi(\pi, \eta)$ decay rates, and the electromagnetic mass differences of the π and K .

³ DONOGHUE 92 result is from a combined analysis of meson masses, $\eta \rightarrow 3\pi$ using second-order chiral perturbation theory including nonanalytic terms, and $(\psi(2S) \rightarrow J/\psi(1S)\pi)/(\psi(2S) \rightarrow J/\psi(1S)\eta)$.

⁴ GERARD 90 uses large N and η - η' mixing.

⁵ LEUTWYLER 90B determines quark mass ratios using second-order chiral perturbation theory for the meson and baryon masses, including nonanalytic corrections. Also uses Weinberg sum rules to determine L_7 .

 m_s/\bar{m} MASS RATIO

$$\bar{m} \equiv (m_u + m_d)/2$$

<u>VALUE</u>	<u>DOCUMENT ID</u>	<u>TECN</u>
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27.3^{+0.7}_{-1.3} OUR EVALUATION See the ideogram below.

$27.35 \pm 0.05^{+0.10}_{-0.07}$ 1 BAZAVOV 14A LATT

26.66 ± 0.32 2 CARRASCO 14 LATT

27.36 ± 0.54 3 ARTHUR 13 LATT

$27.53 \pm 0.20 \pm 0.08$ 4 DURR 11 LATT

• • • We do not use the following data for averages, fits, limits, etc. • • •

26.8 ± 1.4 5 AOKI 11A LATT

27.3 ± 0.9 6 BLOSSIER 10 LATT

28.8 ± 1.65 7 ALLTON 08 LATT

$27.3 \pm 0.3 \pm 1.2$ 8 BLOSSIER 08 LATT

23.5 ± 1.5 9 OLLER 07A THEO

27.4 ± 0.4 10 AUBIN 04 LATT

¹ BAZAVOV 14A is a lattice computation using 4 dynamical flavors of HISQ fermions.

² CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

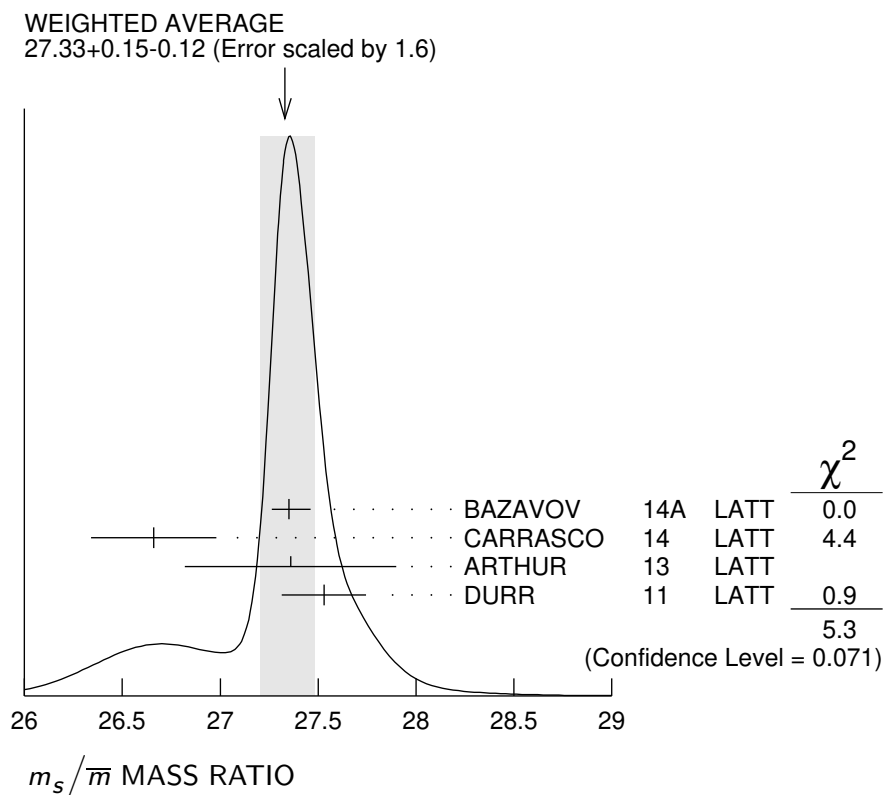
³ ARTHUR 13 is a lattice computation using 2+1 dynamical domain wall fermions.

⁴ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.

⁵ AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using $N_f = 2 + 1$ dynamical flavors of domain wall fermions.

⁶ BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using $N_f=2$ dynamical twisted-mass Wilson fermions.

- ⁷ ALLTON 08 use a lattice computation of the π , K , and Ω masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.
- ⁸ BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.
- ⁹ OLLER 07A use unitarized chiral perturbation theory to order p^4 .
- ¹⁰ Three flavor dynamical lattice calculation of pseudoscalar meson masses.



Q MASS RATIO

$$Q \equiv \sqrt{(m_s^2 - \bar{m}^2)/(m_d^2 - m_u^2)}; \quad \bar{m} \equiv (m_u + m_d)/2$$

VALUE	DOCUMENT ID	TECN
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• • • We do not use the following data for averages, fits, limits, etc. • • •

22.1±0.7	1 COLANGELO 18	THEO
22.0±0.7	2 COLANGELO 17	THEO
21.6±1.1	3 GUO 17	THEO
23.4±0.4±0.5	4 FODOR 16	LATT
21.4±0.4	5 GUO 15F	THEO
22.8±0.4	6 MARTEMYA... 05	THEO
22.7±0.8	7 ANISOVICH 96	THEO

¹ COLANGELO 18 obtain Q from a dispersive analysis of $\eta \rightarrow 3\pi$ decay.

² COLANGELO 17 obtain Q from a dispersive analysis of KLOE collaboration data on $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays and chiral perturbation theory input.

³ GUO 17 determine Q from a dispersive model fit to KLOE and WASA-at-COSY data on $\eta \rightarrow \pi^+ \pi^- \pi^0$ decay and matching to chiral perturbation theory .

⁴ FODOR 16 is a lattice simulation with $N_f = 2 + 1$ dynamical flavors and includes partially quenched QED effects.

⁵ GUO 15F determine Q from a Khuri-Treiman analysis of $\eta \rightarrow 3\pi$ decays.

⁶ MARTEMYANOV 05 determine Q from $\eta \rightarrow 3\pi$ decay.⁷ ANISOVICH 96 find Q from $\eta \rightarrow \pi^+\pi^-\pi^0$ decay using dispersion relations and chiral perturbation theory.**LIGHT QUARKS (u, d, s) REFERENCES**

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COLANGELO	18	EPJ C78 947	G. Colangelo <i>et al.</i>
LYTLE	18	PR D98 014513	A.T. Lytle <i>et al.</i> (HPQCD Collab.)
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FODOR	16	PRL 117 082001	Z. Fodor <i>et al.</i> (BMW Collab.)
BASAK	15	JPCS 640 012052	S. Basak <i>et al.</i> (MILC Collab.)
CHAKRABOR...	15	PR D91 054508	B. Chakraborty <i>et al.</i> (HPQCD Collab.)
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AOKI	11A	PR D83 074508	Y. Aoki <i>et al.</i> (RBC-UKQCD Collab.)
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DOMINGUEZ...	08B	PL B660 49	A. Dominguez-Clarimon, E. de Rafael, J. Taron
ISHIKAWA	08	PR D78 011502	T. Ishikawa <i>et al.</i> (CP-PACS and JLQCD Collabs.)
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OLLER	07A	EPJ A34 371	J.A. Oller, L. Roca
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GOCKELER	06	PR D73 054508	M. Gockeler <i>et al.</i> (QCDSF and UKQCD Collabs)
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PDG	06	JP G33 1	W.-M. Yao <i>et al.</i> (PDG Collab.)
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AOKI	03	PR D67 034503	S. Aoki <i>et al.</i> (CP-PACS Collab.)
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