31. Accelerator Physics of Colliders

Revised August 2019 by M.J. Syphers (Northern Illinois U.; FNAL) and F. Zimmermann (CERN).

This article provides background for the High-Energy Collider Parameter Tables that follow and some additional information.

31.1 Luminosity

The number of events, N_{exp}, is the product of the cross section of interest, σ_{exp}, and the time integral over the instantaneous *luminosity*, $\mathcal{L}$:

$$N_{exp} = \sigma_{exp} \times \int \mathcal{L}(t)dt. \tag{31.1}$$

Today's colliders all employ bunched beams. If two bunches containing n_1 and n_2 particles collide head-on with average collision frequency $f_{\rm coll}$, a basic expression for the luminosity is

$$\mathcal{L} = f_{\rm coll}\frac{n_1 n_2}{4\pi\sigma_x^*\sigma_y^*}\mathcal{F} \tag{31.2}$$

where σ_x^* and σ_y^* characterize the rms transverse beam sizes in the horizontal (bend) and vertical directions at the interaction point, and $\mathcal{F}$ is a factor of order 1, that takes into account geometric effects such as a crossing angle and finite bunch length, and dynamic effects, such as the mutual focusing of the two beam during the collision. For a circular collider, $f_{\rm coll}$ equals the number of bunches per beam times the revolution frequency. In 31.2, it is assumed that the bunches are identical in transverse profile, that the profiles are Gaussian and independent of position along the bunch, and the particle distributions are not altered during bunch crossing. Nonzero beam crossing angles θ_c in the horizontal plane and long bunches (rms bunch length σ_z) will reduce the luminosity, e.g., by a factor $\mathcal{F} \approx 1/(1+\phi^2)^{1/2}$, where the parameter $\phi \equiv \theta_c\sigma_z/(2\sigma_x^*)$ is known as the Piwinski angle. Another luminosity reduction for long bunches is due to the "hourglass" effect (see below).

Whatever the distribution at the source, by the time the beam reaches high energy, the normal form is a useful approximation as suggested by the σ-notation. In the case of an electron storage ring, synchrotron radiation leads to a Gaussian distribution in equilibrium, but even in the absence of radiation the central limit theorem of probability and the diminished importance of space charge effects produce a similar result. Beam tails are often modelled by the superposition of a second Gaussian distribution with larger size and much lower intensity.

The luminosity may be obtained directly by measurement of the beam properties in Eq. 31.2. For continuous measurements, an expression similar to Eq. 31.1 with N_{ref} from a known reference cross section, σ_{ref}, may be used to determine σ_{exp} according to $\sigma_{exp} = (N_{exp}/N_{ref})\sigma_{ref}$.

In the Tables, luminosity is stated in units of cm^{-2}s^{-1}. Integrated luminosity, on the other hand, is usually quoted as the inverse of the standard measures of cross section such as femtobarns and, recently, attobarns. Subsequent sections in this report briefly expand on the dynamics behind collider design, comment on the realization of collider performance in a selection of today's facilities, and end with some remarks on future possibilities.

31.2 Beam Dynamics

The first concern of beam dynamics is stability. While a reference particle proceeds along the design, or reference, trajectory other particles in the bunch are to remain close by. Assume that the reference particle carries a right-handed Cartesian coordinate system, with the z-coordinate pointed in the direction of motion along the reference trajectory. The right-handed coordinate system would indicate the reference particle to travel clockwise around a storage ring. The independent variable is the distance s of the reference particle along this trajectory rather than time, and for simplicity this path is taken to be planar. The transverse coordinates are x and y, where $\{x, z\}$ defines the plane of the reference trajectory.

Several time scales are involved, and the approximations used in writing the equations of motion reflect that circumstance. All of today's high energy colliders are alternating-gradient synchrotrons or, respectively, storage rings [1, 2], and the shortest time scale is that associated with transverse motion, that is described in terms of betatron oscillations, so called because of their analysis for the betatron accelerator species years ago. The linearized equations of motion of a particle displaced from the reference particle are

$$\begin{gathered} x'' + K_x x = 0, \quad K_x \equiv \frac{q}{p}\frac{\partial B}{\partial x} + \frac{1}{\rho^2} \\ y'' + K_y y = 0, \quad K_y \equiv -\frac{q}{p}\frac{\partial B}{\partial x} \\ z' = -x/\rho \end{gathered} \tag{31.3}$$

where the magnetic field $B(s)$ along the design trajectory is only in the y direction, contains only dipole and quadrupole terms, and is treated as static here. The radius of curvature due to the field on the reference orbit is ρ; z represents the longitudinal distance from the reference particle; p and q are the particle's momentum and charge, respectively. The prime denotes d/ds. The pair $(x,\ x')$ describes approximately-canonical variables. For more general cases (e.g. acceleration) one should use $(x,\ p_x)$ instead, where p_x denotes the transverse momentum in the x-direction.

The equations for x and y are those of harmonic oscillators but with a restoring force periodic in s; that is, they are instances of Hill's equation. The solution may be written in the form

$$x(s) = A_x\sqrt{\beta_x}\ \cos\psi_x \tag{31.4}$$

$$x'(s) = -\frac{A_x}{\sqrt{\beta_x}}\left[\alpha_x\cos\psi_x + \sin\psi_x\right] \tag{31.5}$$

where A_x is a constant of integration, $\alpha_x \equiv -(1/2)d\beta_x(s)/ds$, and the envelope of the motion is modulated by the *amplitude function*, β_x. A solution of the same form describes the motion in y. The subscripts will be suppressed in the following discussion.

The amplitude function satisfies

$$2\beta\beta'' - \beta'^2 + 4\beta^2 K = 4, \tag{31.6}$$

and in a region free of magnetic field it should be noted that the solution of Eq. 31.6 is a parabola. Expressing A in terms of $x,\ x'$ yields

$$\begin{aligned} A^2 &= \gamma x^2 + 2\alpha x x' + \beta x'^2 \\ &= \tfrac{1}{\beta}\left[x^2 + (\alpha x + \beta x')^2\right] \end{aligned} \tag{31.7}$$

with $\gamma \equiv (1+\alpha^2)/\beta$. In a single pass system such as a linac, the *Courant-Snyder parameters* α, β, γ may be selected to match the x, x' distribution of the input beam; in a recursive system, the parameters are usually defined by the structure rather than by the beam.

The relationships between the parameters and the structure may be seen by treatment of a simple *lattice* consisting of equally-spaced thin-lens quadrupoles whose magnetic-field gradients are equal in magnitude but alternating in sign. For this discussion, the weak focusing effects of the bending magnets may be neglected. The propagation of $X \equiv \{x,\ x'\}$ through a repetition period may be written $X_2 = MX_1$, with the matrix $M = FODO$ composed of the matrices

$$F = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix},\ D = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix},\ O = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}, \tag{31.8}$$

where f is the magnitude of the focal length and L the lens spacing. Then

$$M = \begin{pmatrix} 1+\frac{L}{f} & 2L+\frac{L^2}{f} \\ -\frac{L}{f^2} & 1-\frac{L}{f}-\frac{L^2}{f^2} \end{pmatrix} . \tag{31.9}$$

The matrix for y is identical in form differing only by a change in sign of the terms linear in $1/f$. An eigenvector-eigenvalue analysis of the matrix M shows that the motion is stable provided $f > L/2$. While that criterion is easily met, in practice instability may be caused by many other factors, including the beam-beam interaction.

Standard focus-drift-defocus-drift, or *FODO*, cells such as characterized in simple form by Eq. 31.9 occupy most of the layout of a large collider ring and may be used to set the scale of the amplitude function and related phase advance. Conversion of Eq. 31.4 to a matrix form equivalent to Eq. 31.9 (but more generally valid, i.e. for any stable periodic linear motion) gives

$$M = \begin{pmatrix} C+\alpha S & \beta S \\ -\gamma S & C-\alpha S \end{pmatrix} \tag{31.10}$$

where $C \equiv \cos\mathit{\Delta}\psi$, $S \equiv \sin\mathit{\Delta}\psi$, and the relation between structure and amplitude function is specified by setting the values of the latter to be the same at both ends of the cell. By comparison of Eq. 31.9 and Eq. 31.10 one finds $C = 1-L^2/(2f^2)$, so that the choice $f = L/\sqrt{2}$ would give a phase advance $\mathit{\Delta}\psi$ of 90 degrees for the standard cell. The amplitude function would have a maximum at the focusing quadrupole of magnitude $\hat{\beta} = 2.7L$, illustrating the relationship of alternating gradient focusing amplitudes to relatively local aspects of the design. Other functionalities such as injection, extraction, and HEP experiments are included by lattice sections matched to the standard cell parameters (β, α) at the insertion points.

The phase advances according to $d\psi/ds = 1/\beta$; that is, β also plays the role of a local $\lambda/2\pi$, and the *tune*, ν, is the number of such oscillations per turn about the closed path. In the neighborhood of an interaction point (IP), the beam optics of the ring is configured so as to produce a narrow focus; the value of the amplitude function at this point is designated β^*.

The motion as it develops with s describes an ellipse in $\{x, x' \equiv dx/ds\}$ phase space, the area of which is πA^2, where A is the constant in Eq. 31.4. If the interior of that ellipse is populated by an ensemble of non-interacting particles, that area, given the name *emittance* and denoted by ε, would change only with energy. More precisely, for a beam with a Gaussian distribution in x, x', the area containing one standard deviation σ_x, divided by π, is used as the definition of emittance in the Tables:

$$\varepsilon_x \equiv \frac{\sigma_x^2}{\beta_x} , \tag{31.11}$$

with a corresponding expression in the other transverse direction, y. For most of the entries in the Tables the standard deviation is used as the beam radius.

At larger transverse amplitudes, due to the influence of nonlinear magnetic fields, the particle motion does not remain linear. Nonlinear fields arise, e.g., from the sextupole magnets deployed to correct the chromaticty (i.e., the change of focusing with particle momentum), or from persistent-current field errors in the superconducting magnets of hadron synchrotrons. At a certain amplitude the nonlinear particle motion ceases to the stable, and particles are lost after circulating for a possibly large number of turns. This limit of stability is called the "dynamic aperture".

To complete the coordinates used to describe the particle motion, and to characterize the longitudinal behavior, we take as the variable conjugate to z the fractional momentum deviation $\delta p/p$ from that of the reference particle. Radiofrequency electric fields in the s direction provide a means for longitudinal oscillations, and the frequency determines the bunch length. The frequency of this system appears in the Tables as does the rms value of $\delta p/p$ characterized as "energy spread" of the beam.

For HEP bunch length is a significant quantity for a variety of reasons, but in the present context if the bunch length, or (with nonzero crossing angle) the effective interaction length, becomes larger than β^* the luminosity is adversely affected. This is because β grows parabolically as one proceeds away from the interaction point and so the beam size increases thus lowering the contribution to the luminosity from such locations. This is often called the "hourglass" effect.

In a storage ring, the bunch length tends to increase with higher bunch intensity due to the so-called longitudinal "impedance" or "wake fields". Similar collective electromagnetic interactions with the beam environment can lead to longitudinal and transverse single- or multi-bunch instabilities [3,4], which may either increase energy spread or beam emittance, and even result in a beam loss.

Another major external electromagnetic field interaction, in the single particle context, is the production of synchrotron radiation due to centripetal acceleration, given by the Larmor formula multiplied by a relativistic magnification factor of γ^4 [5]. In the case of electron rings this process determines the equilibrium emittance through a balance between radiation damping and excitation of oscillations, and further serves as a barrier to future higher energy versions in this variety of collider. A more comprehensive discussion of betatron oscillations, longitudinal motion, and synchrotron radiation is available in the 2008 version of the PDG review [6].

Synchrotron radiation emitted during the collision in the field of the opposing beam is called beamstrahlung. Beamstrahlung is relevant for both linear colliders, where it may degrade the luminosity spectrum, and for future highest-energy circular colliders, where it may limit the beam lifetime, and also increases the energy spread and bunch length of the stored beam. For both types of colliders the beamstrahlung is mitigated by making the colliding beams as flat as possible at the interaction point ($\sigma_x^* \gg \sigma_y^*$). The photon energy spectrum of the beamstrahlung is characterized by the parameter Upsilon $\Upsilon = (2/3)\hbar\omega_c/E_b$ [7], with $\hbar\omega_c$ denoting the critical photon energy and E_b the beam energy. The spectrum strongly deviates from the classical synchrotron radiation spectrum for Υ approaching 1.

31.3 Road to High Luminosity

Eq. 31.2 can be recast in terms of emittances and amplitude functions as

$$\mathcal{L} = f \frac{n_1 n_2}{4\pi\sqrt{\varepsilon_x\,\beta_x^*\,\varepsilon_y\,\beta_y^*}}\,\mathcal{F}. \tag{31.12}$$

Under the assumption $\mathcal{F} \approx 1$, to achieve high luminosity, all one has to do is make high population bunches of low emittance collide at high frequency at locations where the beam optics provides as low values of the amplitude functions as possible.

Expressions for the reductions due to crossing angle and other effects can be found elsewhere [8]. While there are no fundamental limits to producing luminosity, there are certainly challenges. Here we have space to mention only a few of these. The beam-beam tune shift appears in the Tables. A bunch in beam 1 presents a (nonlinear) lens to a particle in beam 2 resulting in changes to the particle's transverse tune with a range characterized by the (vertical) beam-beam parameter [8]

$$\xi_{y,2} = \frac{m_e r_e q_1 q_2 n_1 \beta_{y,2}^*}{2\pi m_{A,2}\gamma_2\sigma_{y,1}^*(\sigma_{x,1}^* + \sigma_{y,1}^*)} \tag{31.13}$$

where r_e denotes the classical electron radius ($r_e \approx 2.8 \times 10^{-15}$ m), m_e the electron mass, q_1 (q_2) the particle charge of beam 1 (2) in units of the elementary charge, and $m_{A,2}$ the mass of beam-2 particles. The transverse oscillations are susceptible to resonant perturbations from a variety of sources such as imperfections in the magnetic guide field, so that certain values of the tune must be avoided. Accordingly, the tune spread arising from ξ is limited [9–11]. A glance at the Tables shows that electrons are more forgiving than protons thanks to the damping effects of synchrotron radiation; the ξ-values for the former are about an order of magnitude larger than those for protons. In linear colliders, the strength of the collision is measured by the ratio of the rms bunch length σ_z to the approximate (linear, thin-lens) beam-beam focal length. This ratio, called disruption parameter D_y [7], is related to ξ_y via $D_y = 4\pi\sigma_z\xi_y/\beta_y^*$. For hadron colliders, two fundamental luminosity limits are the beam lifetime, determined by burn-off in the collisions, and the radiation from the collision debris, which affects the equipment lifetime.

A subject of present intense interest is the *electron-cloud effect* [12, 13]; actually a variety of related processes come under this heading. They typically involve a buildup of electron density in the vacuum chamber due to emission from the chamber walls stimulated by electrons or photons originating from the beam itself. For instance, there is a process closely resembling the multipacting effects familiar from radiofrequency system commissioning. Low energy electrons are ejected from the walls by photons from positron or proton beam-produced synchrotron radiation. These electrons are accelerated toward a beam bunch, but by the time they reach the center of the vacuum chamber the bunch has gone and so the now-energetic electrons strike the opposite wall to produce more secondaries. These secondaries are now accelerated by a subsequent bunch, and so on. Among the disturbances that this electron accumulation can produce is an enhancement of the tune spread within the bunch; the near-cancellation of bunch-induced electric and magnetic fields is no longer in effect.

If the luminosity of Eq. 31.12 is rewritten in terms of the beam-beam parameter, Eq. 31.13, the emittance itself disappears. However, the emittance must be sufficiently small to realize a desired magnitude of beam-beam parameter, but once ξ_y reaches this limit, further lowering the emittance does not lead to higher luminosity.

For electron synchrotrons and storage rings, radiation damping provides an automatic route to achieve a small emittance. In fact, synchrotron radiation is of key importance in the design and optimization of e^+e^- colliders. While vacuum stability and electron clouds can be of concern in the positron rings, synchrotron radiation along with the restoration of longitudinal momentum by the RF system has the positive effect of generating very small transverse beam sizes and small momentum spread. Further reduction of beam size at the interaction points using standard beam optics techniques and successfully contending with high beam currents has led to record luminosities in these rings. To maximize integrated luminosity the beam can be "topped off" by injecting new particles without removing existing ones – a feature difficult to imitate in hadron colliders.

For hadrons, particularly antiprotons, two inventions have played a prominent role. Stochastic cooling [14] was employed first to prepare beams for the $Sp\bar{p}S$ and subsequently in the Tevatron, and to cool the beams at full energy in RHIC [15–17]. Electron cooling [18] was also used in the Tevatron, RHIC and LHC complexes to great advantage. Further innovations are underway driven by the needs of potential future projects; these are noted in the final section. For future energy-frontier hadron colliders, like the proposed FCC-hh and SPPC, also synchrotron radiation damping becomes an important cooling mechanism.

31.4 Recent High Energy Colliders

Collider accelerator physics of course goes far beyond the elements of the preceding sections. In this and the following section elaboration is made on various issues associated with some of the recently operating colliders, particularly factors which impact integrated luminosity. The various colliders utilizing hadrons each have unique characteristics and are, therefore, discussed separately.

As space is limited, general references are provided where much further information can be obtained. A more complete list of recent colliders and their parameters can be found in the High-Energy Collider Parameters tables.

31.4.1 *Tevatron*

The first synchrotron in history using superconducting magnets, the Tevatron [19], was the highest energy collider for 25 years. Its 4.5 T dipole magnets employed superconducting Nb-Ti cable operating at 4.5 K [20], requiring what was then the world's largest cryogenic system [21]. Tevatron operation was terminated in September 2011, after delivering more than 10 fb^{-1} to the p-$\bar{\text{p}}$ collider experiments CDF and D0. The route to high integrated luminosity in the Tevatron was governed by the antiproton production rate, the turn-around time to produce another store, and the resulting optimization of store time. The antiproton production complex [22] consisted of three 8 GeV $\bar{p}$ accelerators — Accumulator, Debuncher, and Recycler — and employed 25 independent stochastic cooling systems and one high energy electron cooling set-up [23] to accumulate up to record high 25×10^{10} $\bar{p}$ per hour. The proton and antiproton beams in the Tevatron circulated in a single vacuum pipe and thus were placed on separated orbits which wrapped around each other in a helical pattern outside of the interaction regions. As the available aperture was limited, the long-range encounters played an important role. Despite these limitations, a total beam-beam tuneshift parameter of $N_{IP}\xi \approx$0.025-0.03 was achieved, a record for hadron beams [24], where N_{IP} denotes the number of collision points (2 for the Tevatron). Other notable advances at the Tevatron included the first permanent-magnet-based high energy accelerator (Recycler), novel longitudinal beam manipulation techniques such as *slip-stacking* and *momentum mining* [25,26], and the first use of electron lenses [27] for beam collimation and for compensation of long-range beam-beam effects. The Tevatron ultimately achieved luminosities a factor of 430 over its original design specification.

31.4.2 *HERA*

HERA [28], operated between 1992 and 2007, delivered nearly 1 fb^{-1} of integrated luminosity to the electron-proton collider experiments H1 and ZEUS. HERA was the first high-energy lepton-hadron collider, and also the first facility to employ both applications of superconductivity: magnets and accelerating structures. The proton beams of HERA had a maximum energy of 920 GeV. The lepton beams (positrons or electrons) were provided by the existing DESY complex, and were accelerated to 27.5 GeV using conventional magnets. At collision a 4-times higher frequency RF system, compared with the injection RF, was used to generate shorter bunches, thus helping alleviate the hourglass effect at the collision points. The lepton beam naturally would become transversely polarized (within about 40 minutes) and "spin rotators" were implemented on either side of an IP to produce longitudinal polarization at the experiment.

31.4.3 *LEP*

Installed in a tunnel of 27 km circumference, LEP [29] was the largest circular e$^+$e$^-$ collider built so far. LEP was operated from 1989 to 2000 with beam energies ranging from 45.6 to 104.5 GeV and a maximum luminosity of 10^{32} cm^{-2}s^{-1}, at 98 GeV, surpassing all relevant design parameters. Up to about 60 GeV, LEP used resonant depolarization to precisely measure the beam energy [30,31].

31.4.4 *SLC*

Based on an existing 3-km long S-band linac, the SLC [32] was the first and only linear collider. It was operated from 1987 to 1998 with a constant beam energy of 45.6 GeV, up to about 80% electron-beam polarization, quasi-flat beams, a final-focus optics with local chromatic correction based on four interleaved sextupoles and $\beta_y^* \approx 1$ mm. In its last year, SLC achieved a peak luminosity of about 3×10^{30} cm^{-2}s^{-1}, roughly half of the design value.

31.5 Present Collider Facilities

31.5.1 *LHC*

The superconducting Large Hadron Collider [33] presently is the world's highest energy collider. Early operations for HEP were first at 3.5 TeV (in 2011) and then 4 TeV per proton [34] (since 2012), with the beam energy increased to 6.5 TeV in 2015. The current status is best checked at the Web site [35]. In 2017 peak luminosities above 2×10^{34} cm^{-2}s^{-1} (more than twice the design value) have been achieved. To meet its luminosity goals the LHC operates with a high beam current of approximately 0.5 A, leading to stored energies of several hundred MJ per beam. Component protection, beam collimation, and controlled energy deposition were given a high priority. Additionally, at energies of 5-7 TeV per particle, synchrotron radiation moves from being a curiosity to a challenge in a hadron accelerator for the first time. At design beam current the cryogenic system must remove roughly 7 kW due to synchrotron radiation, intercepted at a temperature of about 5–20 K. As the photons are emitted their interactions with the vacuum chamber wall can generate free electrons, with consequent "electron cloud" development. Much care was taken to design a special beam screen for the chamber to mitigate this issue. The two proton beams are contained in separate pipes throughout most of the circumference, and are brought together into a single pipe at the interaction points. The large number of bunches, and subsequent short bunch spacing, would lead to approximately 30 head-on collisions through 120 m of common beam pipe at each IP. Thus, a small crossing angle is employed, which reduces the luminosity by about 15%. Still, the bunches moving in one direction experience multiple long-range encounters with the counter-rotating bunches and the resulting perturbations of the particle motion continues to remain a concern. The luminosity scale is absolutely calibrated by the "van der Meer method" as was invented for the ISR [36], and followed by multiple, redundant luminosity monitors (see for example [37] and references therein). The Tables also show the LHC luminosity performance in Pb-Pb collisions, which for the ATLAS and CMS experiments well exceeded the design value, while for the ALICE [38] experiment, the luminosity is "levelled" near the Pb-Pb design value of 10^{27} cm^{-2}s^{-1}. The LHC can also provide Pb-p collisions as it did in 2013 and 2016, and other ion-ion or ion-proton collisions, at different energies.

In the coming years, an ambitious upgrade program, HL-LHC [39,40], has as its target an order-of-magnitude increase in integrated luminosity through the utilization of Nb_3Sn superconducting magnets, superconducting compact "crab" cavities and luminosity leveling also for ATLAS and CMS as its key ingredients.

31.5.2 e^+e^- *Rings*

Asymmetric energies of the two beams have allowed for the enhancement of B-physics research and for interesting interaction region designs. As the bunch spacing can be quite short, the lepton beams sometimes pass through each other at an angle, which may reduce the luminosity — unless the crossing angle can be taken advantage of. KEKB [41] installed high frequency "crab crossing" schemes to fully restore the geometric overlap of the colliding bunches. It attained over 1 fb^{-1} of integrated luminosity in a single day. A different collision approach, called "crab waist", which relies on special sextupoles together with a large crossing angle, has been successfully implemented at DAΦNE [42]. The crab-wiast collision scheme has also been partly realized at the KEKB upgrade, SuperKEKB, and it has indeed become a key ingredient for all proposed future e$^+$e$^-$ circular colliders. SuperKEKB is aiming for luminosities of 8×10^{35} cm^{-2}s^{-1} [43]. Other e$^+$e$^-$ ring colliders currently in operation are BEPC-II, VEPP-2000 and VEPP-4M [43].

31.5.3 *RHIC*

The Relativistic Heavy Ion Collider [44] employs superconducting magnets, and collides combinations of fully-stripped ions such as H-H (p-p), p-Al, p-Au, d-Au, h-Au, Cu-Cu, Cu-Au, Zr-Zr, Ru-Ru, Au-Au, and U-U over a wide energy range [45]. The high charge per particle (+79 for gold, for instance) makes intra-beam scattering of particles within the bunch a special concern, even for seemingly moderate bunch intensities. In 2012, 3-D stochastic cooling was successfully implemented in RHIC [17] and is now routinely used. With stochastic cooling, steady increases in the bunch intensity, and numerous other upgrades, RHIC now operates at 44 times the Au-Au design average luminosity. Another special feature of accelerating heavy ions in RHIC is that the beams cross the "transition energy" during acceleration – a point where the derivative with respect to momentum of the revolution period is zero. This is more typical of low-energy accelerators, where the necessary phase jump required of the RF system is implemented rapidly and little time is spent near this condition. In the case of RHIC with heavy ions, the superconducting magnets do not ramp very quickly and the period of time spent crossing transition is long and must be dealt with carefully. For p-p operation the RHIC beams are always above their transition energy and so this condition is completely avoided. A RHIC physics program in search of a critical point in the nuclear matter phase diagram required operation below the nominal injection energy, and in order to reach the integrated luminosity goals the first bunched beam electron cooler was successfully commissioned for the lowest energies [46].

RHIC is also unique in its ability to accelerate and collide polarized proton beams. As proton beam polarization must be maintained from its low-energy source, successful acceleration through the myriad of depolarizing resonance conditions in high energy circular accelerators has taken years to accomplish. An energy of 255 GeV per proton with 55% final polarization per beam has been realized. As part of a scheme to compensate the head-on beam-beam effect, electron lenses operated routinely during the polarized proton operation at 100 GeV in 2015 [47].

Collisions between a RHIC proton beam and a electron beam stored in a new ring (eRHIC) is one of the two proposed configuration of a future US electron-ion collider (EIC) for nuclear physics [48], the alternative being the addition of figure-8 hadron and electron storage rings to the CEBAF facility at JLAB (JLEIC).

31.6 Future High Energy Colliders and Prospects

Recent accomplishments of particle physics have been obtained through high-energy and high-intensity experiments using hadron-hadron, lepton-lepton, and lepton-proton colliders. Following the discovery of the Higgs particle at the LHC and in view of ongoing searches for "new physics" and rare phenomena, various options are under discussions and development to pursue future particle-physics research at higher energy and with appropriate luminosity. This is the basis for several new projects, ideas, and R&D activities, which can only briefly be summarized here. Specifically, the following projects are noted: an energy upgrade of the LHC based on 16 T dipole magnets (HE-LHC) [49], two approaches to an electron-positron linear collider [50, 51], larger 100-km circular tunnels supporting e^+e^- collisions up to either 240 [52] or 365 GeV [53] in the centre of mass along with a subsequent 70-140 TeV or 100-TeV proton-proton collider, possible future, or far future, muon-ring colliders [54, 55], and potential use of plasma acceleration and other advanced schemes. Complementary studies are ongoing of a high-energy lepton-hadron collider bringing into collision a 60-GeV electron beam from an energy-recovery linac with the 7 TeV protons circulating in the LHC (LHeC) [56, 57], or, much later, with the 50(35) TeV protons of the 100(70) TeV collider (FCC-eh, SPPC), and of $\gamma\gamma$ collider Higgs factories based on recirculating electron linacs (e.g. SAPPHiRE [58]). Tentative parameters of some of the colliders discussed, or mentioned, in this section are summarized in Table 31.1 and Table 31.2.

31.6.1 *Electron-Positron Linear Colliders*

For more than four decades, efforts have been devoted to develop high-gradient technology e^+e^- colliders in order to overcome the synchrotron radiation limitations of circular e^+e^- machines in

Table 31.1: Tentative parameters of selected future e^+e^- high-energy colliders. Parameters associated with different beam energy scenarios are comma-separated.

	FCC-ee	CEPC	ILC	CLIC
Species	e^+e^-	e^+e^-	e^+e^-	e^+e^-
Beam energy (GeV)	46, 120, 183	46, 120	125, 250	190, 1500
Circumference / Length (km)	97.75	100	20.5, 31	11, 50
Interaction regions	2	2	1	1
Est. integrated luminosity per experiment (ab^{-1}/year)	26, 0.9, 0.17	4, 0.4	0.2, 0.2	0.2, 0.6
Peak luminosity (10^{34}/cm^2/s)	230, 8.5, 1.6	32, 3	1.4, 1.8	1.5, 6
Time between collisions (μs)	0.015, 0.75, 8.5	0.025, 0.68	0.55	0.0005
Energy spread (rms, 10^{-3})	1.3, 1.65, 2.0	0.4, 1.0	e^-: 1.9, 1.2 e^+: 1.5, 0.7	3.5
Bunch length (rms, mm)	12.1, 5.3, 3.8	8.5, 3.3	0.3	0.09, 0.044
IP beam size (μm)	H: 6.3, 14, 38 V: 0.03, 0.04, 0.07	H: 5.9, 21 V: 0.04, 0.07	H: 0.52, 0.47 V: 0.008, 0.006	H: 0.15, 0.04 V: 0.003, 0.001
Injection energy (GeV)	on energy (topping off)	on energy (topping off)	5.0 (linac)	9.0 (linac)
Transv. rms emittance (pm)	H: 270, 630, 1340 V: 1, 1, 3	H: 170, 1210 V: 2, 3	H: 20, 10 V: 0.14, 0.07	H: 2.4, 0.22 V: 0.8, 0.01
β^* at interaction point (cm)	H: 15, 30, 100 V: 0.08, 0.1, 0.16	H: 20, 36 V: 0.1, 0.15	H: 1.3, 2.2 V: 0.041, 0.048	H: 0.8, 0.69 V: 0.01, 0.0068
Full crossing angle (mrad)	30	33	14	20
Crossing scheme	crab waist	crab waist	crab crossing	crab crossing
Piwinski angle $\phi = \sigma_z\theta_c/(2\sigma_x^*)$	28.5, 5.8, 1.5	23.8, 2.6	0	0
Beam-beam param. ξ_y (10^{-3})	133, 118, 144	72, 109	n/a	n/a
Disruption parameter D_y	0.9, 1.1, 1.9	0.3, 1.0	34, 25	8, 12
Average Upsilon Υ	0.0002,0.0004,0.0006	0.0001,0.0005	0.03, 0.06	0.26, 3.4
RF frequency (MHz)	400, 400, 800	650	1300	11994
Particles per bunch (10^{10})	17, 15, 27	8, 15	2	0.52, 0.37
Bunches per beam	16640, 328, 33	12000, 242	1312 (pulse)	352, 312 (trains at 50 Hz)
Average beam current (mA)	1390, 29, 5.4	19.2	6 (in train)	1660, 1200 (in train)
RF gradient (MV/m)	1.3, 9.8, 19.8	3.6, 19.7	31.5	72, 100
Polarization (%)	≥10, 0, 0	5–10, 0	e^-: 80% e^+: 30%	e^-: 70% at IP
SR power loss (MW)	100	64	n/a	n/a
Beam power/beam (MW)	n/a	n/a	5.3, 10.5	3, 14
Novel technology	—	—	high grad. SC RF	two-beam accel.

the TeV energy range.

The primary challenge confronting a high energy, high luminosity single pass collider design is the power requirement, so that measures must be taken to keep the demand within bounds as illustrated in a transformed Eq. 31.2 [60]:

$$\mathcal{L} \approx \frac{137}{8\pi r_e} \frac{P_{\text{wall}}}{E_{cm}} \frac{\eta}{\sigma_y^*} N_\gamma H_D \,. \tag{31.14}$$

Here, P_{wall} is the total wall-plug power of the collider, $\eta \equiv P_b/P_{\text{wall}}$ the efficiency of converting wall-plug power into beam power $P_b = f_{\text{coll}} n E_{cm}$, E_{cm} the cms energy, n $(= n_1 = n_2)$ the bunch population, and σ_y^* the vertical rms beam size at the collision point. In formulating Eq. 31.14 the number of beamstrahlung photons emitted per $e^\pm$, was approximated as $N_\gamma \approx 2\alpha r_e n/\sigma_x^*$, where

Table 31.2: Tentative parameters of selected future high-energy hadronic colliders. Parameters associated with different beam energy scenarios for a μ collider are comma-separated. Parameters of HL-LHC can be found in the High-Energy Collider Parameters review tables. The listed luminosity for the LHeC refers to parasitic operation in parallel to the HL-LHC pp collisions; it could be significantly increased for dedicated operation [59].

	LHeC	HE-LHC	FFC-hh	SPPC	μ collider
Species	ep	pp	pp	pp	$\mu^+\mu^-$
Beam Energy (TeV)	0.06(e), 7 (p)	13.5	50	37.5	0.063, 3
Circumference (km)	9(e), 26.7 (p)	26.7	97.75	100	0.3, 6
Interaction regions	1	2 (4)	4	2	1, 2
Estimated integrated luminosity per experiment (ab^{-1}/year)	0.1	0.5	0.2–1.0	0.4	0.001, 1.0
Peak luminosity (10^{34}/cm^2/s)	0.8	16	5–30	10	2.2, 71
Time between collisions (μs)	0.025	0.025	0.025	0.025	1, 20
Energy spread (rms, 10^{-3})	0.03 (e), 0.1(p)	0.1	0.1	0.2	0.04, 1
Bunch length (rms, mm)	0.06 (e), 75.5(p)	80	80	75.5	63, 2
IP beam size (μm)	4.3 (round)	8.8	6.7-3.5 (init.)	6.8 (init.)	75, 1.5
Injection energy (GeV)	1(e), 450(p)	1300	3300	2100	on energy
Transverse emittance (rms, nm)	0.45(e), 0.27(p)	0.17	0.04 (init.)	0.06 (init.)	335, 0.9
β^*, amplitude fcn. at IP (cm)	5.0(e), 7.0(p)	45	110–30	75	1.7, 0.25
Beam-beam parameter/IP (10^{-3})	$-$(e), 0.4(p)	12	5–15	7.5	20, 90
RF frequency (MHz)	800(e), 400(p)	400	400	400/200	805
Particles per bunch (10^{10})	0.23(e), 22(p)	22	10	15	400, 200
Bunches per beam	$-$(e), 2808(p)	2808	10600	10080	1
Average beam current (mA)	15(e), 883(p)	1120	500	730	640, 16 (peak)
Length of standard cell (m)	52.4(e arc), 107(p)	137	213	148	N/A
Phase advance per cell (deg)	310/90(eH/V) 90(p)	90	90	90	N/A
Peak magnetic field (T)	0.264(e), 8.33(p)	16	16	12	10
Polarization (%)	90(e), 0(p)	0	0	0	0
SR power loss/beam (MW)	30(e), 0.01(p)	0.1	2.4	1.1	3×10^{-5}, 0.068
Novel technology	high-energy ERL	16T Nb$_3$Sn magnets	16T Nb$_3$Sn magnets	HTS magnets	muon prod.

α denotes the fine-structure constant. The management of $P_{\rm wall}$ leads to an upward push on the bunch population n with an attendant rise in the energy radiated due to the electromagnetic field of one bunch acting on the particles of the other. Keeping a significant fraction of the luminosity close to the nominal energy represents a design goal, which is met if N_γ does not exceed a value of about 1. A consequence is the use of flat beams, where N_γ is managed by the beam width, and luminosity adjusted by the beam height, thus the explicit appearance of the vertical beam size σ_y^*. The final factor in Eq. 31.14, H_D, represents the enhancement of luminosity due to the pinch effect during bunch crossing (the effect of which has been neglected in the expression for N_γ).

The approach designated by the International Linear Collider (ILC) is presented in the Tables, and the contrast with the collision-point parameters of the circular colliders is striking, though reminiscent in direction of those of the SLAC Linear Collider. The ILC *Technical Design Report* [50, 61] has a baseline cms energy of 500 GeV with upgrade provision for 1 TeV, and luminosity comparable to the LHC; recent tendancies have been toward a baseline of 250 GeV. The ILC is based on superconducting accelerating structures of the 1.3 GHz TESLA variety. Progress toward higher field gradients and Q values continues to be made, with nitrogen-doping techniques being a recent example [62].

At CERN, a design effort is underway on the Compact Linear Collider (CLIC), each linac of which is itself a two-beam accelerator, in that a high energy, low current beam is fed by a low energy, high current driver [63]. The CLIC design employs normal conducting 12 GHz accelerating structures at a gradient of 100 MeV/m, some three times the current capability of the superconducting ILC cavities. The design cms energy is 3 TeV, though recent staging options – 0.38, 1.5, and 3 TeV – have been developed [51].

31.6.2 *Future Circular Colliders*

The discovery, in 2012, of the Higgs boson at the LHC has stimulated interest in constructing a large circular tunnel which could host a variety of energy-frontier machines, including high-energy electron-positron, proton-proton, and lepton-hadron colliders. Such projects are under study by a global collaboration hosted at CERN (FCC) [53, 64, 65] and another one centered in China (CEPC/SPPC) [52], following earlier proposals for a Very Large Hadron Collider (VLHC) [66] and a Very Large Lepton Collider (VLLC) in the US, which would have been housed in the same 230-km long tunnel.

The maximum beam energy of a hadron collider is directly proportional to the magnetic field and to the ring circumference. The LHC magnets, based on Nb-Ti superconductor, achieve a maximum operational field of 8.33 T. The HL-LHC project develops the technology of higher field Nb_3Sn magnets as well as cables made from high-temperature superconductor (HTS). Nb_3Sn dipoles could ultimately reach an operational field around 16 T, and HTS inserts, requiring new engineering materials and substantial dedicated R&D, could boost this further. More cost-effective hybrid magnet designs incorporating Nb-Ti, two types of Nb_3Sn, and an inner layer of HTS providing fields of about 20 T have been examined [67]. However present project efforts are not utilizing this hybrid approach as of yet.

Aside from the magnets, the cryogenic beam vacuum system is another key component of any future hadron collider. A beam screen inside the cold bore of the magnets can intercept the synchrotron radiation at an elevated temperature, allowing a more efficient extraction of the synchrotron-radiation heat load. While the LHC beam screen has a temperature of 5–20 K, future, higher-energy machines are likely to raise this temperature to 50 K or 100 K.

Further substantial increases in collision energy are possible only with a larger tunnel. The FCC hadron collider (FCC-hh) [64, 68, 69], formerly called VHE-LHC, is based on a new tunnel of about 100 km circumference, which would allow exploring energies up to 100 TeV in the centre of mass with proton-proton collisions, using 16 T magnets. This new tunnel could also accommodate a high-luminosity circular e^+e^- Higgs factory (FCC-ee) as well as a lepton-hadron collider (FCC-eh). The SPPC is a 100 km hadron collider based on 12 T (later 24 T) iron-based high-temperature superconducting magnets, which could be installed in the same tunnel as the e^+e^- collider CEPC.

In order to serve as a Higgs factory a new circular e^+e^- collider needs to achieve a cms energy of at least 240 GeV. FCC-ee [53, 68] (formerly TLEP), installed in the $\sim$100 km tunnel of the FCC-hh, could reach even higher energies, e.g. 365 GeV cms for $t\bar{t}$ production. At these energies, the luminosity, limited by the synchrotron radiation power, would still be above 10^{34} cm^{-2}s^{-1} at each of two or four collision points. At lower energies (Z pole and WW threshold) FCC-ee could deliver two to three orders of magnitude higher luminosities, and also profit from radiative self polarization for precise energy calibration. The short beam lifetime at the high target luminosity, due to radiative Bhabha scattering, requires FCC-ee to be constructed as a double ring, where the collider rings operating at constant energy are complemented by a full-energy injector ring installed in the same tunnel to "top off" the collider current. Beamstrahlung, i.e. synchrotron radiation emitted during the collision in the field of the opposing beam, introduces an additional beam lifetime limitation depending on momentum acceptance (so that achieving sufficient off-momentum dynamic aperture becomes one of the design challenges), as well as some bunch lengthening.

31.6.3 *Muon Collider*

The muon to electron mass ratio of 210 implies less concern about synchrotron radiation by a factor of about 2×10^9 and its 2.2 μs lifetime means that it will last for some $300B$ turns in a ring with an average bending magnets field of B (Tesla). Design effort became serious in the mid 1990s and a collider outline emerged quickly.

Removal of the synchrotron radiation barrier reduces the scale of a muon collider facility to a level compatible with on-site placement at existing accelerator laboratories. The Higgs production cross section in the s-channel is enhanced by a factor of $(m_\mu/m_e)^2$ compared to that in e^+e^- collisions. The obvious advantage in colliding muons rather than protons is that the muon collider center of mass energy $\sqrt{s}$, is entirely available to produce short-distance reactions rather than being spread among proton constituents and, e.g., a 14 TeV muon collider with sufficient luminosity might be very effective as a direct exploration machine, with a physics potential similar to that of a 100 TeV proton-proton collider [70]. Muon colliders are expected to be more compact, power efficient and significantly less expensive than equivalent energy frontier hadron or e^+e^- machines, and a neutrino factory could potentially be realized in the course of construction [71].

The challenges to luminosity achievement are clear and amenable to immediate study: targeting, collection, and emittance reduction are paramount, as well as the bunch manipulation required to produce $> 10^{12}$ muons per bunch without emittance degradation. A multi-TeV c.m.e. high luminosity $O(10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1})$ muon collider would consist of [72,73] : (i) a high power proton driver (SRF 8 GeV 2-4 MW H^- linac), (ii) pre-target accumulation and compressor rings, in which high intensity 1-3 ns long proton bunches are formed, (iii) a liquid-mercury target for converting the proton beam into a tertiary muon beam with energy of about 200 MeV, (iv) a multi-stage ionization cooling section that reduces transverse and longitudinal emittances and creates a low emittance beam, (v) a multistage acceleration system, possibly employing recirculating linear accelerators (RLA) to accelerate muons in a modest number of turns up to 2 TeV using superconducting RF technology, and, finally, (vi) a 2–6 km diameter collider ring located some 100 m underground, where counter-propagating muon beams are stored and collide over the roughly 1000–2000 turns corresponding to the muon lifetime.

Collection of muons from the decay of pions produced in proton-nucleus interactions results in a large initial 6D phase volume for the muons, which must be reduced (cooled) by a factor of 10^6, otherwise, the luminosity reach will not exceed $O(10^{31}\ \mathrm{cm}^{-2}\mathrm{s}^{-1})$. The technique of ionization cooling [74,75] is uniquely applicable to muons because of their minimal interaction with matter [76]; a proof-of-principle was recently demonstrated in the pioneering MICE experiment at RAL [77]. Muon collider R&D has led to a number of remarkable advances in the past decade [78], including a novel concept to generate muon pairs at threshold using the annihilation of 45 GeV positrons with electrons at rest [79,80], and alternative concepts based on laser-hadron collisions [55,81,82], all of which might allow low emittance beams to be obtained directly, without any cooling.

31.6.4 *Plasma Acceleration and Other Advanced Concepts*

At the 1956 CERN Symposium, a paper by Veksler, in which he suggested acceleration of protons to the TeV scale using a bunch of electrons, anticipated current interest in plasma acceleration [83]. A half-century later this became more than a suggestion, with the demonstration, as a striking example, of electron energy doubling from 42 to 84 GeV over 85 cm at SLAC [84], the creation of a 1 GeV electron bunch with relatively small energy spread accelerated through a cm-scale plasma [85], and the achievement of proton-driven plasma acceleration of electrons at CERN [86].

Whether plasma acceleration will find application in an HEP facility is not yet clear, given the necessity of staging and phase-locking acceleration in multiple plasma chambers. However, strides continue to be made, as multi-stage coupling of independent laser plasma accelerators have been demonstrated recently [87]. Another critical issue is the power efficiency η for a collider based on plasma acceleration, whose luminosity would still be described by 31.14. Maintaining beam quality and beam position as well as the acceleration of high-repetition bunch trains are also primary feasibility issues, addressed by active R&D. For a recent status report on laser-plasma acceleration and the steps towards a future electron positron collider based on this technology, see [88].

Additional approaches aiming at accelerating gradients higher, or much higher, than those achievable with conventional metal cavities include the use of dielectric materials and, for the long-term future, crystals. Combining several innovative ideas, even a linear crystal muon collider driven by X-ray lasers has been proposed [89], as well as "accelerators on a chip" [90, 91]. Not only the achievable accelerating gradient, but also the overall power efficiency, e.g. the attainable luminosity as a function of electrical input power, along with the beam stability [92] will determine the suitability of any novel technology for use in future high-energy accelerators.

References

[1] E. D. Courant and H. S. Snyder, Annals Phys. **3**, 1 (1958), Annals Phys.281,360(2000). This is the classic article on the alternating gradient synchrotron.

[2] A. W. Chao *et al.*, editors, *Handbook of accelerator physics and engineering*, World Scientific, Hackensack, USA (2013), ISBN 9789814415842, URL `http://www.worldscientific.com/worldscibooks/10.1142/8543`.

[3] A. W. Chao, *Physics of collective beam instabilities in high energy accelerators*, Wiley, New York, NY (1993), URL `https://cds.cern.ch/record/246480`.

[4] K. Y. Ng, *Physics of intensity dependent beam instabilities*, World Scientific, Hoboken, NJ (2006), URL `https://cds.cern.ch/record/1012829`.

[5] H. Wiedemann, in *Handbook of Accelerator Physics and Engineering* [2], *ibid*, Sec. 3.1.

[6] C. Amsler *et al.* (Particle Data Group), Phys. Lett. **B667**, 1 (2008).

[7] K. Yokoya and P. Chen, Lect. Notes Phys. **400**, 415 (1992).

[8] M.A. Furman and M.S. Zisman, *Handbook of Accelerator Physics and Engineering* [2], *ibid*, Sec. 4.1.

[9] R. Assmann and K. Cornelis, in "Particle accelerator. Proceedings, 7th European Conference, EPAC 2000, Vienna, Austria, June 26-30, 2000. Vol. 1-3," 1187–1189 (2000), URL `http://weblib.cern.ch/abstract?CERN-SL-2000-046-OP`.

[10] J. Gao, Nucl. Instrum. Meth. **A533**, 270 (2004).

[11] K. Ohmi and F. Zimmermann, Phys. Rev. ST Accel. Beams **18**, 12, 121003 (2015).

[12] M.A. Furman, *Handbook of Accelerator Physics and Engineering* [2], *ibid*, Sec. 2.4.14.

[13] http://ab-abp-rlc.web.cern.ch/ab-abp-rlc-ecloud/. This site contains many references as well as videos of electron cloud simulations.

[14] D. Mohl *et al.*, Phys. Rept. **58**, 73 (1980).

[15] M. Blaskiewicz and J. M. Brennan, Phys. Rev. ST Accel. Beams **10**, 061001 (2007).

[16] M. Blaskiewicz, J. M. Brennan and F. Severino, Phys. Rev. Lett. **100**, 174802 (2008).

[17] M. Blaskiewicz, J. M. Brennan and K. Mernick, Phys. Rev. Lett. **105**, 094801 (2010).

[18] G. I. Budker, Sov. Atom. Energ. **22**, 438 (1967), [At. Energ.22,346(1967)].

[19] H. T. Edwards, Ann. Rev. Nucl. Part. Sci. **35**, 605 (1985).

[20] A. Tollestrup and E. Todesco, in "Reviews Of Accelerator Science And Technology: Volume 1," 185–210, World Scientific (2008).

[21] W. Fowler, Particle Accelerators **26**, 179 (1990).

[22] M. Church and J. Marriner, Annual Review of Nuclear and Particle Science **43**, 1, 253 (1993).

[23] S. Nagaitsev *et al.*, Physical Review Letters **96**, 4, 044801 (2006).

[24] V. Shiltsev *et al.*, Physical Review Special Topics-Accelerators and Beams **8**, 10, 101001 (2005).

[25] K. Koba and J. Steimel, in "AIP Conference Proceedings," volume 642, 223–225, AIP (2002).

[26] C. Bhat, Physics Letters A **330**, 6, 481 (2004).

[27] V. D. Shiltsev, *Electron lenses for super-colliders*, Springer (2016).

[28] Brief history at http://en.wikipedia.org/wiki/Hadron_ Elektron_Ring_Anlage .

[29] R. Assmann, M. Lamont and S. Myers, Nucl. Phys. Proc. Suppl. **109B**, 17 (2002), [,17(2002)].

[30] R. Assmann *et al.*, Eur. Phys. J. **C6**, 187 (1999).

[31] R. Assmann *et al.*, in "Proceedings, 1999 Particle Accelerator Conference (PAC'99): New York, New York, March 29-April 2, 1999," 2999–3001 (1999), URL `http://accelconf.web.cern.ch/AccelConf/p99/PAPERS/THP23.PDF`.

[32] N. Phinney, eConf **C00082**, MO102 (2000), [,1(2000)], [arXiv:physics/0010008].

[33] L. Evans, Ann. Rev. Nucl. Part. Sci. **61**, 435 (2011).

[34] M. Draper, editor, *Proceedings, LHC Performance Workshop (Chamonix 2014)*, CERN, CERN, Geneva (2015), ISBN 9789290834137.

[35] Detailed information from the multi-volume design report to present status may be found at http://lhc.web.cern.ch/lhc/.

[36] S. van der Meer (1968).

[37] G. Aad *et al.* (ATLAS), Eur. Phys. J. **C73**, 8, 2518 (2013), [arXiv:1302.4393].

[38] http://aliceinfo.cern.ch/Public/Welcome.html.

[39] http://hilumilhc.web.cern.ch.

[40] G. Apollinari *et al.*, CERN Yellow Rep. Monogr. **4**, 1 (2017).

[41] S. Kurokawa and E. Kikutani, Nucl. Instrum. Meth. **A499**, 1 (2003).

[42] M. Zobov *et al.*, Phys. Rev. Lett. **104**, 174801 (2010).

[43] An overview of electron-positron colliders past and present may be found in ICFA Beam Dynamics Newsletter No. 46, April 2009, http://icfa-bd.kek.jp/. A daily account of the luminosity progress at SuperKEKB may be found at http://www-linac.kek.jp/skekb/snapshot/dailysnap.html.

[44] M. Harrison *et al.*, Nucl. Instrum. Meth. **A499**, 1 (2003).

[45] http://www.rhichome.bnl.gov/RHIC/Runs/.

[46] A. Fedotov, et al., Proceedings NAPAC 2019 (2019).

[47] W. Fischer *et al.*, Phys. Rev. Lett. **115**, 26, 264801 (2015).

[48] F. Pilat, in "Proceedings, 9th International Particle Accelerator Conference (IPAC 2018): Vancouver, BC Canada," TUXGBD3 (2018).

[49] F. Zimmermann *et al.*, The European Physical Journal Special Topics **228**, 5, 1109 (2019), ISSN 1951-6401, URL `https://doi.org/10.1140/epjst/e2019-900088-6`.

[50] P. Bambade *et al.* (2019), [arXiv:1903.01629].

[51] T. K. Charles *et al.* (CLICdp, CLIC), CERN Yellow Rep. Monogr. **1802**, 1 (2018), [arXiv:1812.06018].

[52] CEPC Study Group (2018), [arXiv:1809.00285].

[53] M. Benedikt *et al.*, The European Physical Journal Special Topics **228**, 2, 261 (2019).

[54] D. Neuffer and V. Shiltsev, JINST **13**, 10, T10003 (2018), [arXiv:1811.10694].

[55] F. Zimmermann MOPMF065 (2018), [J. Phys. Conf. Ser.1067,no.2,022017(2018)].

[56] J. L. Abelleira Fernandez *et al.* (LHeC Study Group), J. Phys. **G39**, 075001 (2012), [arXiv:1206.2913].

[57] J. L. Abelleira Fernandez *et al.* (LHeC Study Group) (2012), [arXiv:1211.5102].

[58] S. A. Bogacz *et al.* (2012), [arXiv:1208.2827].

[59] F. Bordry *et al.* (2018), [arXiv:1810.13022].

[60] F. Zimmermann, AIP Conf. Proc. **592**, 1, 494 (2001).

[61] `http://www.linearcollider.org/ILC/Publications/ Technical-Design-Report`.

[62] A. Grassellino *et al.*, Supercond. Sci. Technol. **26**, 102001 (2013), [arXiv:1306.0288].

[63] http://lcd.web.cern.ch/lcd/CDR/CDR.html.

[64] M. Benedikt *et al.*, The European Physical Journal Special Topics **228**, 4, 755 (2019), ISSN 1951-6401, URL `https://doi.org/10.1140/epjst/e2019-900087-0`.

[65] M. Mangano *et al.*, The European Physical Journal C **79**, 6, 474 (2019), ISSN 1434-6052, URL `https://doi.org/10.1140/epjc/s10052-019-6904-3`.

[66] http://vlhc.org.

[67] E. Todesco and F. Zimmermann, editors, *Proceedings, EuCARD-AccNet-EuroLumi Workshop: The High-Energy Large Hadron Collider (HE-LHC10): Villa Bighi, Malta, Republic of Malta, October 14-16, 2010. EuCARD-AccNet-EuroLumi Workshop: The High-Energy Large Hadron Collider*, CERN, CERN, Geneva (2011), 29 lectures, 156 pages, published as CERN Yellow Report, URL `https://cds.cern.ch/record/1344820`.

[68] http://fcc.web.cern.ch.

[69] M. Benedikt, D. Schulte and F. Zimmermann, Phys. Rev. ST Accel. Beams **18**, 101002 (2015).

[70] D. Neuffer and V. Shiltsev, Journal of Instrumentation **13**, 10, T10003 (2018).

[71] http://en.wikipedia.org/wiki/Neutrino_Factory.

[72] C. M. Ankenbrandt *et al.*, Physical Review Special Topics-Accelerators and Beams **2**, 8, 081001 (1999).

[73] J.-P. Delahaye *et al.*, in "Proceedings, 2013 Community Summer Study on the Future of U.S. Particle Physics: Snowmass on the Mississippi (CSS2013): Minneapolis, MN, USA, July 29-August 6, 2013," (2013), [arXiv:1308.0494], URL `http://www.slac.stanford.edu/econf/C1307292/docs/submittedArxivFiles/1308.0494.pdf`.

[74] A. Skrinsky and V. Parkhomchuk, Sov. J. Part. Nucl **12**, 223, 210 (1981).

[75] D. Neuffer, Particle Accelerators **14**, 75 (1983).

[76] R. Palmer, Reviews of Accelerator Science and Technology **7**, 137 (2014).

[77] M. Bogomilov *et al.*, arXiv preprint arXiv:1907.08562 (2019).

[78] M. Boscolo, J. P. Delahaye and M. Palmer, Reviews of Accelerator Science and Technology **10**, 189 (2019), arXiv preprint arXiv:1808.01858.

[79] M. Antonelli *et al.*, Nucl. Instrum. Meth. **A807**, 101 (2016), [arXiv:1509.04454].

[80] M. Biagini *et al.*, in "Proceedings, 10th International Particle Accelerator Conference (IPAC2019): Melbourne, Australia, May 19-24, 2019," MOZZPLS2 (2019).

[81] C. Curatolo, F. Broggi and L. Serafini, Nucl. Instrum. Meth. **A865**, 128 (2017).

[82] M. W. Krasny (2015), [arXiv:1511.07794].

[83] V. I. Veksler (1956), URL `http://cds.cern.ch/record/1241563`.

[84] I. Blumenfeld *et al.*, Nature **445**, 741 (2007).

[85] W. P. Leemans *et al.*, Nature Phys. **2**, 696 (2006).

[86] E. Gschwendtner and M. Turner (AWAKE) (2019), [arXiv:1901.04171].

[87] S. Steinke *et al.*, Nature **530**, 7589, 190 (2016).

[88] ALEGRO Collab., Towards an Advanced Linear International Collider, arXiv:1901.10370 .

[89] V. D. Shiltsev, Phys. Usp. **55**, 965 (2012), [arXiv:1205.3087].

[90] J. Breuer and P. Hommelhoff, Phys. Rev. Lett. **111**, 13, 134803 (2013), [arXiv:1308.0464].

[91] E. A. Peralta *et al.*, Nature **503**, 91 (2013).

[92] V. Lebedev, A. Burov and S. Nagaitsev, Phys. Rev. Accel. Beams **20**, 12, 121301 (2017), [Erratum: Phys. Rev. Accel. Beams **21**, 059901 (2018)], [arXiv:1701.01498].