

75.  $B^0-\bar{B}^0$  Mixing

Revised March 2020 by O. Schneider (EPFL).

There are two neutral  $B^0-\bar{B}^0$  meson systems,  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  (generically denoted  $B_q^0-\bar{B}_q^0$ ,  $q = s, d$ ), which exhibit particle-antiparticle mixing [1]. This mixing phenomenon is described in Ref. [2]. In the following, we adopt the notation introduced in Ref. [2], and assume  $CPT$  conservation throughout. In each system, the light (L) and heavy (H) mass eigenstates,

$$|B_{L,H}\rangle = p|B_q^0\rangle \pm q|\bar{B}_q^0\rangle, \quad (75.1)$$

have a mass difference  $\Delta m_q = m_H - m_L > 0$ , a total decay width difference  $\Delta\Gamma_q = \Gamma_L - \Gamma_H$  and an average decay width  $\Gamma_q = (\Gamma_L + \Gamma_H)/2$ . In the absence of  $CP$  violation in the mixing,  $|q/p| = 1$ , the differences are given by  $\Delta m_q = 2|M_{12}|$  and  $|\Delta\Gamma_q| = 2|\Gamma_{12}|$ , where  $M_{12}$  and  $\Gamma_{12}$  are the off-diagonal elements of the mass and decay matrices [2]. The evolution of a pure  $|B_q^0\rangle$  or  $|\bar{B}_q^0\rangle$  state at  $t = 0$  is given by

$$|B_q^0(t)\rangle = g_+(t)|B_q^0\rangle + \frac{q}{p}g_-(t)|\bar{B}_q^0\rangle, \quad (75.2)$$

$$|\bar{B}_q^0(t)\rangle = g_+(t)|\bar{B}_q^0\rangle + \frac{p}{q}g_-(t)|B_q^0\rangle, \quad (75.3)$$

which means that the flavor states remain unchanged (+) or oscillate into each other (−) with time-dependent probabilities proportional to

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma_q t}}{2} \left[ \cosh\left(\frac{\Delta\Gamma_q}{2}t\right) \pm \cos(\Delta m_q t) \right]. \quad (75.4)$$

In the absence of  $CP$  violation, the time-integrated mixing probability  $\int |g_-(t)|^2 dt / (\int |g_-(t)|^2 dt + \int |g_+(t)|^2 dt)$  is given by

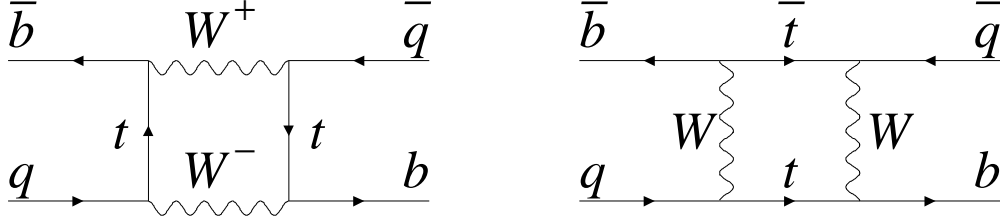
$$\chi_q = \frac{x_q^2 + y_q^2}{2(x_q^2 + 1)}, \quad \text{where} \quad x_q = \frac{\Delta m_q}{\Gamma_q}, \quad y_q = \frac{\Delta\Gamma_q}{2\Gamma_q}. \quad (75.5)$$

## 75.1 Standard Model predictions and phenomenology

In the Standard Model, the transitions  $B_q^0 \rightarrow \bar{B}_q^0$  and  $\bar{B}_q^0 \rightarrow B_q^0$  are due to the weak interaction. They are described, at the lowest order, by box diagrams involving two  $W$  bosons and two up-type quarks (see Fig. 75.1), as is the case for  $K^0 - \bar{K}^0$  mixing. However, the long range interactions arising from intermediate virtual states are negligible for the neutral  $B$  meson systems, because the large  $B$  mass is off the region of hadronic resonances. The calculation of the dispersive and absorptive parts of the box diagrams yields the following predictions for the off-diagonal element of the mass and decay matrices [3],

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_B m_{B_q} B_{B_q} f_{B_q}^2}{12\pi^2} S_0(m_t^2/m_W^2) (V_{tq}^* V_{tb})^2, \quad (75.6)$$

$$\begin{aligned} \Gamma_{12} = & \frac{G_F^2 m_b^2 \eta'_B m_{B_q} B_{B_q} f_{B_q}^2}{8\pi} \\ & \times \left[ (V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{cb} \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) \right. \\ & \left. + (V_{cq}^* V_{cb})^2 \mathcal{O}\left(\frac{m_c^4}{m_b^4}\right) \right], \quad (75.7) \end{aligned}$$



**Figure 75.1:** Dominant box diagrams for the  $B_q^0 \rightarrow \bar{B}_q^0$  transitions ( $q = d$  or  $s$ ). Similar diagrams exist where one or both  $t$  quarks are replaced with  $c$  or  $u$  quarks.

where  $G_F$  is the Fermi constant,  $m_W$  the  $W$  boson mass, and  $m_i$  the mass of quark  $i$ ;  $m_{B_q}$ ,  $f_{B_q}$  and  $B_{B_q}$  are the  $B_q^0$  mass, weak decay constant and bag parameter, respectively. The known function  $S_0(x_t)$  can be approximated very well by  $0.784 x_t^{0.76}$  [4], and  $V_{ij}$  are the elements of the CKM matrix [5]. The QCD corrections  $\eta_B$  and  $\eta'_B$  are of order unity. The only non-negligible contributions to  $M_{12}$  are from box diagrams involving two top quarks. The phases of  $M_{12}$  and  $\Gamma_{12}$  satisfy

$$\phi_M - \phi_\Gamma = \pi + \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right), \quad (75.8)$$

implying that the mass eigenstates have mass and width differences of opposite signs. This means that, like in the  $K^0-\bar{K}^0$  system, the heavy state is expected to have a smaller decay width than that of the light state:  $\Gamma_H < \Gamma_L$ . Hence,  $\Delta\Gamma_q = \Gamma_L - \Gamma_H$  is expected to be positive in the Standard Model.

Furthermore, the quantity

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq \frac{3\pi}{2} \frac{m_b^2}{m_W^2} \frac{1}{S_0(m_t^2/m_W^2)} \sim \mathcal{O}\left(\frac{m_b^2}{m_t^2}\right) \quad (75.9)$$

is small, and a power expansion of  $|q/p|^2$  yields

$$\left|\frac{q}{p}\right|^2 = 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\phi_M - \phi_\Gamma) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right). \quad (75.10)$$

Therefore, considering both Eqs. (75.8) and (75.9), the  $CP$ -violating parameter

$$1 - \left|\frac{q}{p}\right|^2 \simeq \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \quad (75.11)$$

is expected to be very small:  $\sim \mathcal{O}(10^{-3})$  for the  $B_d^0-\bar{B}_d^0$  system and  $\lesssim \mathcal{O}(10^{-4})$  for the  $B_s^0-\bar{B}_s^0$  system [6].

In the approximation of negligible  $CP$  violation in mixing, the ratio  $\Delta\Gamma_q/\Delta m_q$  is equal to the small quantity  $|\Gamma_{12}/M_{12}|$  of Eq. (75.9); it is hence independent of CKM matrix elements, *i.e.*, the same for the  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  systems. Calculations [7] yield  $\sim 5 \times 10^{-3}$  with a  $\sim 20\%$  uncertainty. Given the published experimental knowledge [8] on the mixing parameter  $x_q$

$$\begin{cases} x_d = 0.769 \pm 0.004 & (B_d^0-\bar{B}_d^0 \text{ system}) \\ x_s = 26.89 \pm 0.07 & (B_s^0-\bar{B}_s^0 \text{ system}) \end{cases}, \quad (75.12)$$

the Standard Model thus predicts that  $\Delta\Gamma_d/\Gamma_d$  is very small (below 1%), but  $\Delta\Gamma_s/\Gamma_s$  considerably larger ( $\sim 10\%$ ). These width differences are caused by the existence of final states to which both the  $B_q^0$  and  $\bar{B}_q^0$  mesons can decay. Such decays involve  $b \rightarrow c\bar{c}q$  quark-level transitions, which are Cabibbo-suppressed if  $q = d$  and Cabibbo-allowed if  $q = s$ .

A complete set of Standard Model predictions for all mixing parameters in both the  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  systems can be found in Ref. [9].

## 75.2 Experimental issues and methods for oscillation analyses

Time-integrated measurements of  $B^0-\bar{B}^0$  mixing were published for the first time in 1987 by UA1 [10] and ARGUS [11], and since then by many other experiments. These measurements are typically based on counting same-sign and opposite-sign lepton pairs from the semileptonic decay of the produced  $b\bar{b}$  pairs. Such analyses cannot easily separate the contributions from the different  $b$ -hadron species, therefore, the clean environment of  $\Upsilon(4S)$  machines (where only  $B_d^0$  and charged  $B_u$  mesons are produced) is in principle best suited to measure  $\chi_d$ .

However, better sensitivity is obtained from time-dependent analyses aiming at the direct measurement of the oscillation frequencies  $\Delta m_d$  and  $\Delta m_s$ , from the proper time distributions of  $B_d^0$  or  $B_s^0$  candidates identified through their decay in (mostly) flavor-specific modes, and suitably tagged as mixed or unmixed. This is particularly true for the  $B_s^0-\bar{B}_s^0$  system, where the large value of  $x_s$  implies maximal mixing, *i.e.*,  $\chi_s \simeq 1/2$ . In such analyses, the  $B_d^0$  or  $B_s^0$  mesons are either fully reconstructed, partially reconstructed from a charm meson, selected from a lepton with the characteristics of a  $b \rightarrow \ell^-$  decay, or selected from a reconstructed displaced vertex. At high-energy colliders (LEP, SLC, Tevatron, LHC), the proper time  $t = \frac{m_B}{p}L$  is measured from the distance  $L$  between the production vertex and the  $B$  decay vertex, and from an estimate of the  $B$  momentum  $p$ . At asymmetric  $B$  factories (KEKB, PEP-II), producing  $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_d^0 \bar{B}_d^0$  events with a boost  $\beta\gamma$  ( $= 0.425, 0.55$ ), the proper time difference between the two  $B$  candidates is estimated as  $\Delta t \simeq \frac{\Delta z}{\beta\gamma c}$ , where  $\Delta z$  is the spatial separation between the two  $B$  decay vertices along the boost direction. In all cases, the good resolution needed on the vertex positions is obtained with silicon detectors.

The average statistical significance  $\mathcal{S}$  of a  $B_q^0$  oscillation signal can be approximated as [12]

$$\mathcal{S} \approx \sqrt{N/2} f_{\text{sig}} (1 - 2\eta) e^{-(\Delta m_q \sigma_t)^2/2}, \quad (75.13)$$

where  $N$  is the number of selected and tagged candidates,  $f_{\text{sig}}$  is the fraction of signal in that sample,  $\eta$  is the total mistag probability, and  $\sigma_t$  is the resolution on proper time (or proper time difference). The quantity  $\mathcal{S}$  decreases very quickly as  $\Delta m_q$  increases; this dependence is controlled by  $\sigma_t$ , which is therefore a critical parameter for  $\Delta m_s$  analyses. At high-energy colliders, the proper time resolution  $\sigma_t \sim \frac{m_B}{\langle p \rangle} \sigma_L \oplus t \frac{\sigma_p}{p}$  includes a constant contribution due to the decay length resolution  $\sigma_L$  (typically 0.04–0.3 ps), and a term due to the relative momentum resolution  $\sigma_p/p$  (typically 10–20% for partially reconstructed decays), which increases with proper time. At  $B$  factories, the boost of the  $B$  mesons is estimated from the known beam energies, and the term due to the spatial resolution dominates (typically 1–1.5 ps because of the much smaller  $B$  boost).

In order to tag a  $B_q^0$  candidate as mixed or unmixed, it is necessary to determine its flavor both in the initial state and in the final state. The initial and final state mistag probabilities,  $\eta_i$  and  $\eta_f$ , degrade  $\mathcal{S}$  by a total factor  $(1 - 2\eta) = (1 - 2\eta_i)(1 - 2\eta_f)$ . In lepton-based analyses, the final state is tagged by the charge of the lepton from  $b \rightarrow \ell^-$  decays; the largest contribution to  $\eta_f$  is then due to  $\bar{b} \rightarrow \bar{c} \rightarrow \ell^-$  decays. Alternatively, the charge of a reconstructed charm meson ( $D^{*-}$  from  $B_d^0$  or  $D_s^-$  from  $B_s^0$ ), or that of a kaon hypothesized to come from a  $b \rightarrow c \rightarrow s$  decay [13], can be used. For fully-inclusive analyses based on topological vertexing, final-state tagging techniques include

jet-charge [14] and charge-dipole [15, 16] methods. At high-energy colliders, the methods to tag the initial state (*i.e.*, the state at production), can be divided into two groups: the ones that tag the initial charge of the  $\bar{b}$  quark contained in the  $B_q^0$  candidate itself (same-side tag), and the ones that tag the initial charge of the other  $b$  quark produced in the event (opposite-side tag). On the same side, the sign of a charged pion, kaon or proton from the primary vertex is correlated with the production state of the  $B_q^0$  meson if that particle is a decay product of a  $B^{**}$  state or the first in the fragmentation chain [17, 18]. Jet- and vertex-charge techniques work on both sides and on the opposite side, respectively. Finally, the charge of a lepton from  $b \rightarrow \ell^-$ , of a kaon from  $b \rightarrow c \rightarrow s$  or of a charm hadron from  $b \rightarrow c$  [19] can be used as an opposite-side tag, keeping in mind that its performance is degraded due to integrated mixing. At SLC, the beam polarization produced a sizeable forward-backward asymmetry in the  $Z \rightarrow b\bar{b}$  decays, and provided another very interesting and effective initial state tag based on the polar angle of the  $B_q^0$  candidate [15]. Initial state tags have also been combined to reach  $\eta_i \sim 26\%$  at LEP [18, 20] or 22% at SLD [15] with full efficiency. In the case  $\eta_f = 0$ , this corresponds to an effective tagging efficiency  $Q = \epsilon D^2 = \epsilon(1 - 2\eta)^2$ , where  $\epsilon$  is the tagging efficiency, in the range 23 – 31%. The equivalent figure achieved by CDF during Tevatron Run I was  $\sim 3.5\%$  (see tagging summary on page 160 of Ref. [21]), reflecting the fact that tagging is more difficult at hadron colliders. The CDF and DØ analyses of Tevatron Run II data reached  $\epsilon D^2 = (1.8 \pm 0.1)\%$  [22] and  $(2.5 \pm 0.2)\%$  [23] for opposite-side tagging, while same-side kaon tagging (for  $B_s^0$  analyses) contributed an additional 3.7 – 4.8% at CDF [22], and pushed the combined performance to  $(4.7 \pm 0.5)\%$  at DØ [24]. LHCb, operating in the forward region at the LHC where the environment is different in terms of track multiplicity and  $b$ -hadron production kinematics, has reported  $\epsilon D^2 = (2.10 \pm 0.25)\%$  [25] for opposite-side tagging,  $(1.80 \pm 0.26)\%$  [26] for same-side kaon tagging, and  $(2.11 \pm 0.11)\%$  [27] for same-side pion and proton tagging: the combined figure ranges typically between  $(3.73 \pm 0.15)\%$  [28] and  $(5.33 \pm 0.25)\%$  [29] depending on the mode in which the tagged  $B_s^0$  meson is reconstructed, and reaches up to  $(8.1 \pm 0.6)\%$  [30] for hadronic  $B_d^0$  modes.

At  $B$  factories, the flavor of a  $B_d^0$  meson at production cannot be determined, since the two neutral  $B$  mesons produced in a  $\Upsilon(4S)$  decay evolve in a coherent  $P$ -wave state where they keep opposite flavors at any time. However, as soon as one of them decays, the other follows a time-evolution given by Eqs. (75.2) or (75.3), where  $t$  is replaced with  $\Delta t$  (which will take negative values half of the time). Hence, the “initial state” tag of a  $B$  can be taken as the final-state tag of the other  $B$ . Effective tagging efficiencies of 30% are achieved by BaBar and Belle [31], using different techniques including  $b \rightarrow \ell^-$  and  $b \rightarrow c \rightarrow s$  tags. It is worth noting that, in this case, mixing of the other  $B$  (*i.e.*, the coherent mixing occurring before the first  $B$  decay) does not contribute to the mistag probability.

Before the experimental observation of a decay-width difference, oscillation analyses typically neglected  $\Delta\Gamma_q$  in Eq. (75.4), and described the time dependence with the functions  $\Gamma_q e^{-\Gamma_q t} (1 \pm \cos(\Delta m_q t))/2$  (high-energy colliders) or  $\Gamma_d e^{-\Gamma_d |\Delta t|} (1 \pm \cos(\Delta m_d \Delta t))/4$  (asymmetric  $\Upsilon(4S)$  machines). As can be seen from Eq. (75.4), a non-zero value of  $\Delta\Gamma_q$  would effectively reduce the oscillation amplitude with a small time-dependent factor that would be very difficult to distinguish from time resolution effects. Measurements of  $\Delta m_q$  are usually extracted from the data using a maximum likelihood fit.

### 75.3 $\Delta m_d$ and $\Delta\Gamma_d$ measurements

Many  $B_d^0-\bar{B}_d^0$  oscillations analyses have been published [32] by the ALEPH [33], DELPHI [16, 34], L3 [35], OPAL [36, 37], BaBar [38], Belle [39], CDF [17], DØ [23], and LHCb [40–43] collaborations.

Although a variety of different techniques have been used, the individual  $\Delta m_d$  results obtained

at LEP and Tevatron have remarkably similar precision. Their average is compatible with the recent and more precise measurements at the asymmetric  $B$  factories and the LHC. The systematic uncertainties are not negligible; they are often dominated by sample composition, mistag probability, or  $b$ -hadron lifetime contributions. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the  $b$ -hadron lifetimes and fractions published in this *Review*. Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fragmentation fractions, lifetimes, branching ratios of  $b$  hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, tagging, background description). Combining all measurements [16, 17, 23, 33–43] and accounting for all identified correlations yields  $\Delta m_d = 0.5065 \pm 0.0016(\text{stat}) \pm 0.0011(\text{syst}) \text{ ps}^{-1}$  [8], a result dominated by the latest LHCb measurement with  $B^0 \rightarrow D^{(*)-} \mu^+ \nu_\mu X$  decays [43].

On the other hand, ARGUS and CLEO have published time-integrated measurements [44–46], which average to  $\chi_d = 0.182 \pm 0.015$ . Following Ref. [46], the width difference  $\Delta\Gamma_d$  could in principle be extracted from the measured value of  $\Gamma_d$  and the above averages for  $\Delta m_d$  and  $\chi_d$  (see Eq. (75.5)), provided that  $\Delta\Gamma_d$  has a negligible impact on the  $\Delta m_d$  measurements. However, direct time-dependent studies published by DELPHI [16], BaBar [47], Belle [48], LHCb [49], ATLAS [50] and CMS [51] provide stronger constraints, which can be combined to yield [8]

$$\Delta\Gamma_d/\Gamma_d = +0.001 \pm 0.010. \quad (75.14)$$

Assuming  $\Delta\Gamma_d = 0$  and no  $CP$  violation in mixing, and using the  $B_d^0$  lifetime average of  $1.519 \pm 0.004 \text{ ps}$  [8], the  $\Delta m_d$  and  $\chi_d$  results are combined to yield the world average

$$\Delta m_d = 0.5065 \pm 0.0019 \text{ ps}^{-1} \quad (75.15)$$

or, equivalently,

$$\chi_d = 0.1858 \pm 0.0011. \quad (75.16)$$

This  $\Delta m_d$  value provides an estimate of  $2|M_{12}|$ , and can be used with Eq. (75.6) to extract  $|V_{td}|$  within the Standard Model [52]. The main experimental uncertainties on the result come from  $m_t$  and  $\Delta m_d$ , but are still completely negligible with respect to the uncertainty due to the hadronic matrix element  $f_{B_d} \sqrt{B_{B_d}} = 225 \pm 9 \text{ MeV}$  [53] obtained from three-flavor lattice QCD calculations.

#### 75.4 $\Delta m_s$ and $\Delta\Gamma_s$ measurements

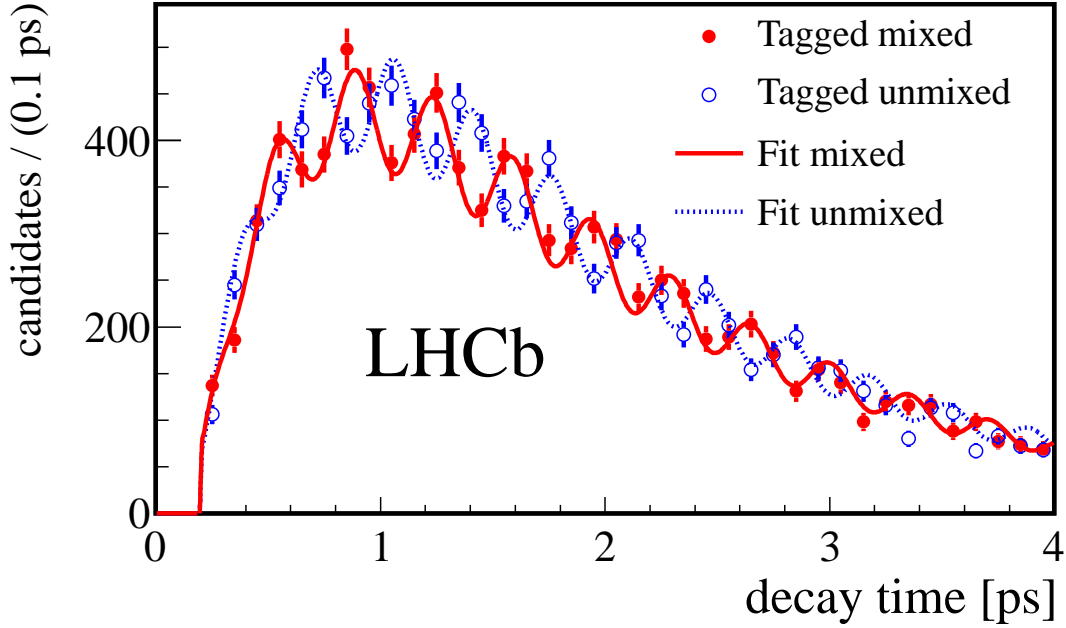
After many years of intense search at LEP and SLC,  $B_s^0-\bar{B}_s^0$  oscillations were first observed in 2006 by CDF using  $1 \text{ fb}^{-1}$  of Tevatron Run II data [22]. More recently LHCb observed  $B_s^0-\bar{B}_s^0$  oscillations independently with  $B_s^0 \rightarrow D_s^- \pi^+$  [40, 54],  $B_s^0 \rightarrow D_s^- \mu^+ \nu X$  [42] and even  $B_s^0 \rightarrow J/\psi K^+ K^-$  [28] decays, using between 1 and  $4.9 \text{ fb}^{-1}$  of data collected at the LHC until 2016. Taking systematic correlations into account, the average of all published measurements of  $\Delta m_s$  [22, 28, 40, 42, 54] is

$$\Delta m_s = 17.749 \pm 0.019(\text{stat}) \pm 0.007(\text{syst}) \text{ ps}^{-1}, \quad (75.17)$$

dominated by LHCb (see Fig. 75.2) and still statistically limited.

The information on  $|V_{ts}|$  obtained in the framework of the Standard Model is hampered by the hadronic uncertainty, as in the  $B_d^0$  case. However, several uncertainties cancel in the frequency ratio

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2, \quad (75.18)$$



**Figure 75.2:** Proper time distribution of  $B_s^0 \rightarrow D_s^- \pi^+$  candidates tagged as mixed (red) or unmixed (blue) in the LHCb experiment, displaying  $B_s^0-\bar{B}_s^0$  oscillations (from Ref. [54]).

where the SU(3) flavor-symmetry breaking factor  $\xi = (f_{B_s} \sqrt{B_{B_s}})/(f_{B_d} \sqrt{B_{B_d}}) = 1.206 \pm 0.017$  is obtained from a combination of three-flavor lattice QCD calculations [53] dominated by the results of Ref. [55]. Using the measurements of Eqs. (75.15) and (75.17), one can extract

$$\left| \frac{V_{td}}{V_{ts}} \right| = 0.2054 \pm 0.0004(\text{exp}) \pm 0.0029(\text{lattice}), \quad (75.19)$$

in good agreement with (but much more precise than) the value obtained from the ratio of the  $b \rightarrow d\gamma$  and  $b \rightarrow s\gamma$  transition rates observed at the  $B$  factories [52].

The CKM matrix can be constrained using experimental results on observables such as  $\Delta m_d$ ,  $\Delta m_s$ ,  $|V_{ub}/V_{cb}|$ ,  $\epsilon_K$ , and  $\sin(2\beta)$  together with theoretical inputs and unitarity conditions [52,56,57]. The constraint from our knowledge on the ratio  $\Delta m_s/\Delta m_d$  is more effective in limiting the position of the apex of the CKM unitarity triangle than the one obtained from the  $\Delta m_d$  measurements alone, due to the reduced hadronic uncertainty in Eq. (75.18). We also note that the measured value of  $\Delta m_s$  is consistent with the Standard Model prediction obtained from CKM fits where no experimental information on  $\Delta m_s$  is used, *e.g.*,  $17.25 \pm 0.85 \text{ ps}^{-1}$  [56] or  $16.70_{-0.45}^{+0.73} \text{ ps}^{-1}$  [57].

Information on  $\Delta\Gamma_s$  can be obtained from the study of the proper time distribution of untagged  $B_s^0$  samples [58]. In the case of an inclusive  $B_s^0$  selection [59], or a flavor-specific (semileptonic or hadronic)  $B_s^0$  decay selection [20,60–62], both the short- and long-lived components are present, and the proper time distribution is a superposition of two exponentials with decay constants  $\Gamma_{L,H} = \Gamma_s \pm \Delta\Gamma_s/2$ . In principle, this provides sensitivity to both  $\Gamma_s$  and  $(\Delta\Gamma_s/\Gamma_s)^2$ . Ignoring  $\Delta\Gamma_s$  and fitting for a single exponential leads to an estimate of  $1/\Gamma_s$  (called effective lifetime) with a relative bias proportional to  $(\Delta\Gamma_s/\Gamma_s)^2$ . An alternative approach, sensitive to first order in  $\Delta\Gamma_s/\Gamma_s$ , is to determine the effective lifetime of untagged  $B_s^0$  decays to pure  $CP$  eigenstates; measurements exist for  $B_s^0 \rightarrow D_s^+ D_s^-$  [61],  $B_s^0 \rightarrow K^+ K^-$  [62,63],  $B_s^0 \rightarrow J/\psi \eta$  [64],  $B_s^0 \rightarrow J/\psi f_0(980)$  [65],  $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$  [51,66],  $B_s^0 \rightarrow J/\psi K_S^0$  [67], and  $B_s^0 \rightarrow \mu^+ \mu^-$  [68]. The extraction of  $1/\Gamma_s$  and  $\Delta\Gamma_s$  from such measurements, discussed in detail in Ref. [69], requires additional information in

the form of theoretical assumptions or external inputs on weak phases and hadronic parameters. In what follows, we only use the effective lifetimes of decays to  $CP$ -even ( $D_s^+ D_s^-$ ,  $J/\psi\eta$ ) and  $CP$ -odd ( $J/\psi f_0(980)$ ,  $J/\psi\pi^+\pi^-$ ) final states where  $CP$  conservation can be assumed.

The best sensitivity to  $1/\Gamma_s$  and  $\Delta\Gamma_s$  is achieved by the time-dependent measurements of the  $B_s^0 \rightarrow J/\psi K^+ K^-$  (including  $B_s^0 \rightarrow J/\psi\phi$ ) and  $B_s^0 \rightarrow \psi(2S)\phi$  decay rates performed at CDF [70], DØ [71], ATLAS [72, 73], CMS [74] and LHCb [28, 75, 76], where the  $CP$ -even and  $CP$ -odd amplitudes are separated statistically through a full angular analysis. The LHCb collaboration analyzes the  $B_s^0 \rightarrow J/\psi K^+ K^-$  decay considering that the  $K^+ K^-$  system can be in a P-wave or S-wave state, and measures the dependence of the strong phase difference between the P-wave and S-wave amplitudes as a function of the  $K^+ K^-$  invariant mass [28, 77]; this allows the unambiguous determination of the sign of  $\Delta\Gamma_s$ , which is found to be positive. All these studies use both untagged and tagged  $B_s^0$  candidates and are optimized for the measurement of the  $CP$ -violating phase  $\phi_s^{c\bar{c}s}$ , defined as the weak phase difference between the  $B_s^0-\bar{B}_s^0$  mixing amplitude and the  $b \rightarrow c\bar{c}s$  decay amplitude. As reported below in Eq. (75.28), the current experimental average of  $\phi_s^{c\bar{c}s}$  is consistent with zero. Assuming no  $CP$  violation (*i.e.*,  $\phi_s^{c\bar{c}s} = 0$ ) a combination [8] of the published  $B_s^0 \rightarrow J/\psi K^+ K^-$ ,  $J/\psi\phi$  and  $\psi(2S)\phi$  analyses [28, 70–72, 74–76] and of effective lifetime measurements with flavor-specific [20, 60–62] and pure  $CP$  [51, 61, 64–66] final states yields

$$\Delta\Gamma_s = +0.085 \pm 0.004 \text{ ps}^{-1} \text{ and } 1/\Gamma_s = 1.515 \pm 0.004 \text{ ps}, \quad (75.20)$$

or, equivalently,

$$1/\Gamma_L = 1.423 \pm 0.005 \text{ ps} \quad \text{and} \quad 1/\Gamma_H = 1.620 \pm 0.007 \text{ ps}, \quad (75.21)$$

in good agreement with the Standard Model prediction  $\Delta\Gamma_s = 0.088 \pm 0.020 \text{ ps}^{-1}$  [9].

Estimates of  $\Delta\Gamma_s/\Gamma_s$  obtained from measurements of the  $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$  branching fractions are not included in the average, since they are based on the questionable [7] assumption that these decays account for all  $CP$ -even final states.

### 75.5 Average $b$ -hadron mixing probability and $b$ -hadron production fractions at high energy

Mixing measurements can significantly improve our knowledge on the fractions  $f_u$ ,  $f_d$ ,  $f_s$ , and  $f_{\text{baryon}}$ , defined as the fractions of  $B_u$ ,  $B_d^0$ ,  $B_s^0$ , and  $b$ -baryons in an unbiased sample of weakly-decaying  $b$  hadrons produced in high-energy collisions. Indeed, time-integrated mixing analyses using lepton pairs from  $b\bar{b}$  events at high energy measure the quantity

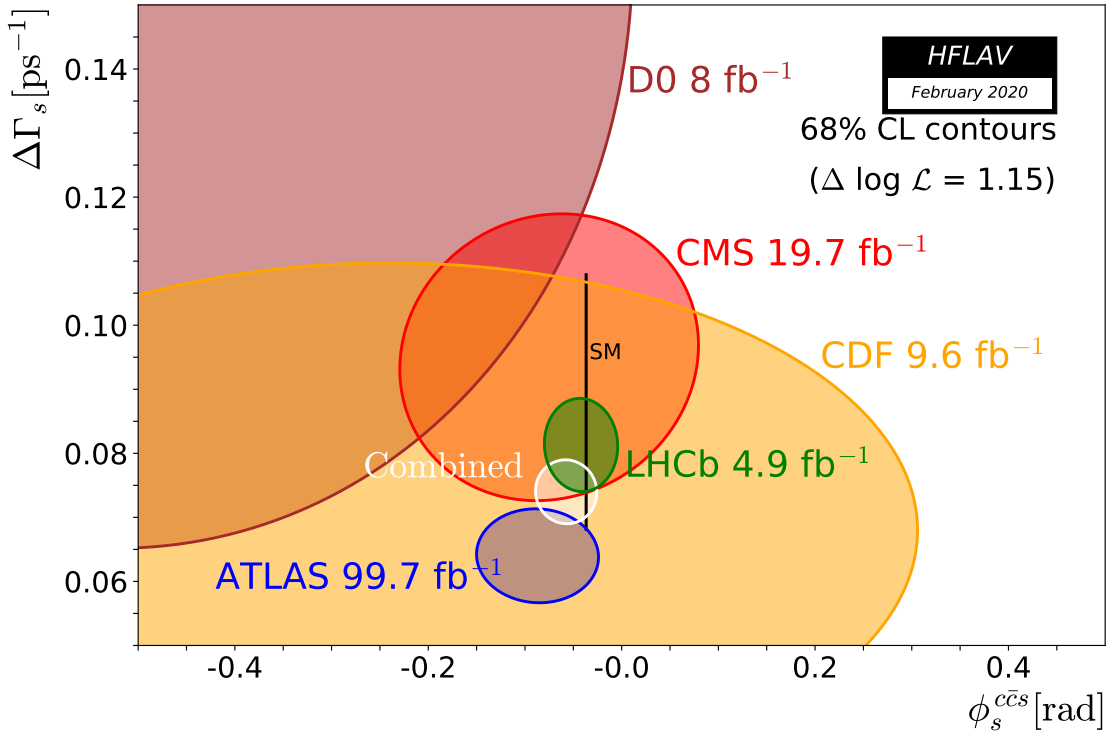
$$\bar{\chi} = f'_d \chi_d + f'_s \chi_s, \quad (75.22)$$

where  $f'_q$  ( $q = s, d$ ) is the  $B_q^0$  fraction in a sample of semileptonic  $b$ -hadron decays. Assuming that all  $b$  hadrons have the same semileptonic decay width implies  $f'_q = f_q/(\Gamma_q\tau_b)$ , where  $\tau_b$  is the average  $b$ -hadron lifetime. Hence  $\bar{\chi}$  measurements performed at LEP [78] and Tevatron [79, 80], together with  $\chi_d$  given in Eq. (75.16) and the very good approximation  $\chi_s = 1/2$  (in fact  $\chi_s = 0.499312 \pm 0.000004$  from Eqs. (75.5), (75.17) and (75.20)), provide constraints on  $f_d$  and  $f_s$ .

The LEP experiments have measured  $\mathcal{B}(\bar{b} \rightarrow B_s^0) \times \mathcal{B}(B_s^0 \rightarrow D_s^- \ell^+ \nu_\ell X)$  [81],  $\mathcal{B}(b \rightarrow \Lambda_b^0) \times \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \ell^- \bar{\nu}_\ell X)$  [82], and  $\mathcal{B}(b \rightarrow \Xi_b^-) \times \mathcal{B}(\Xi_b^- \rightarrow \Xi^- \ell^- \bar{\nu}_\ell X)$  [83] from partially reconstructed final states including a lepton,  $f_{\text{baryon}}$  from protons identified in  $b$  events [84], and the production rate of charged  $b$  hadrons [85]. The  $b$ -hadron fraction ratios measured at CDF are based on double semileptonic  $K^* \mu \mu$  and  $\phi \mu \mu$  final states [86] and lepton-charm final states [87]; in addition CDF and DØ have both measured strange  $b$ -baryon production [88]. On the other hand, fraction ratios have been studied by LHCb using fully reconstructed hadronic  $B_s^0$  and  $B_d^0$  decays [89], as well as

**Table 75.1:**  $\bar{\chi}$  and  $b$ -hadron fractions (see text).

	in $Z$ decays [8]	at Tevatron [8]	at LHC [89–91]
$\bar{\chi}$	$0.1259 \pm 0.0042$	$0.147 \pm 0.011$	
$f_u = f_d$	$0.408 \pm 0.007$	$0.344 \pm 0.021$	
$f_s$	$0.100 \pm 0.008$	$0.115 \pm 0.013$	
$f_{\text{baryon}}$	$0.084 \pm 0.011$	$0.198 \pm 0.046$	
$f_s/f_d$	$0.246 \pm 0.023$	$0.333 \pm 0.040$	$0.247 \pm 0.009$



**Figure 75.3:** 68% CL contours in the  $(\phi_s^{c\bar{c}s}, \Delta\Gamma_s)$  plane, showing all measurements from CDF [70], DØ [71], ATLAS [72, 73], CMS [74] and LHCb [28, 29, 75, 76, 93], with their preliminary average [8]. The very thin black rectangle represents the Standard Model predictions of  $\phi_s^{c\bar{c}s}$  [57] and  $\Delta\Gamma_s$  [9].

semileptonic decays of  $\Lambda_b^0$ ,  $B_s^0$ ,  $B_d^0$  and  $B_u$  [90]. ATLAS has measured  $f_s/f_d$  using  $B_s^0 \rightarrow J/\psi\phi$  and  $B^0 \rightarrow J/\psi K^{*0}$  decays [91]. Both CDF and LHCb observe that the ratio  $f_{\Lambda_b^0}/(f_u + f_d)$  decreases with the transverse momentum of the lepton+charm system, indicating that the  $b$ -hadron fractions are not the same in different environments. A combination of the available information from LEP and Tevatron yields, under the constraints  $f_u = f_d$ ,  $f_u + f_d + f_s + f_{\text{baryon}} = 1$  and Eq. (75.22), the averages of the first two columns of Table 75.1, while the third column shows the average of LHC measurements of  $f_s/f_u = f_s/f_d$ , which are all compatible. The  $B_c^+$  fraction, neglected in the above constraints, has been measured for the first time by LHCb to be  $(0.26 \pm 0.06)\%$  [92].

### 75.6 $CP$ -violation studies

Evidence for  $CP$  violation in  $B_q^0-\bar{B}_q^0$  mixing has been searched for, both with flavor-specific and inclusive  $B_q^0$  decays, in samples where the initial flavor state is tagged, usually with a lepton from



the other  $b$ -hadron in the event. In the case of semileptonic (or other flavor-specific) decays, where the final-state tag is also available, the following asymmetry [2]

$$\mathcal{A}_{\text{SL}}^q = \frac{N(\bar{B}_q^0(t) \rightarrow \ell^+ \nu_\ell X) - N(B_q^0(t) \rightarrow \ell^- \bar{\nu}_\ell X)}{N(\bar{B}_q^0(t) \rightarrow \ell^+ \nu_\ell X) + N(B_q^0(t) \rightarrow \ell^- \bar{\nu}_\ell X)} \simeq 1 - |q/p|_q^2 \quad (75.23)$$

has been measured either in time-integrated analyses at CLEO [46, 94], BaBar [95], CDF [96], DØ [97–99] and LHCb [100], or in time-dependent analyses at LEP [37, 101], BaBar [47, 102] and Belle [103]. In the inclusive case, also investigated at LEP [101, 104], no final-state tag is used, and the asymmetry [105]

$$\begin{aligned} & \frac{N(\bar{B}_q^0(t) \rightarrow \text{all}) - N(B_q^0(t) \rightarrow \text{all})}{N(\bar{B}_q^0(t) \rightarrow \text{all}) + N(B_q^0(t) \rightarrow \text{all})} \\ & \simeq \mathcal{A}_{\text{SL}}^q \left[ \sin^2 \left( \frac{\Delta m_q t}{2} \right) - \frac{x_q}{2} \sin(\Delta m_q t) \right] \end{aligned} \quad (75.24)$$

must be measured as a function of the proper time to extract information on  $CP$  violation. In addition LHCb has studied the time dependence of the charge asymmetry of  $B^0 \rightarrow D^{(*)-} \mu^+ \nu_\mu X$  decays without tagging the initial state [106], which would be equal to

$$\frac{N(D^{(*)-} \mu^+ \nu_\mu X) - N(D^{(*)+} \mu^- \bar{\nu}_\mu X)}{N(D^{(*)-} \mu^+ \nu_\mu X) + N(D^{(*)+} \mu^- \bar{\nu}_\mu X)} = \mathcal{A}_{\text{SL}}^d \frac{1 - \cos(\Delta m_d t)}{2} \quad (75.25)$$

in absence of detection and production asymmetries.

The DØ collaboration measured a like-sign dimuon charge asymmetry in semileptonic  $b$  decays that deviates by  $2.8\sigma$  from the tiny Standard Model prediction and concluded, from a more refined analysis in bins of muon impact parameters, that the overall discrepancy is at the level of  $3.6\sigma$  [97]. In all other cases, asymmetries compatible with zero (and the Standard Model [9]) have been found, with a precision limited by the available statistics. Several of the analyses at high energy don't disentangle the  $B_d^0$  and  $B_s^0$  contributions, and either quote a mean asymmetry or a measurement of  $\mathcal{A}_{\text{SL}}^d$  assuming  $\mathcal{A}_{\text{SL}}^s = 0$ : we no longer include these in the average. An exception is the dimuon DØ analysis [97], which separates the two contributions by exploiting their dependence on the muon impact parameter cut. The resulting measurements of  $\mathcal{A}_{\text{SL}}^d$  and  $\mathcal{A}_{\text{SL}}^s$  are then both compatible with the Standard Model. They are also correlated. We therefore perform a two-dimensional average of the measurements of Refs. [46, 47, 94, 95, 97–100, 102, 103, 106] and obtain [8]

$$\mathcal{A}_{\text{SL}}^d = -0.0021 \pm 0.0017, \text{ or } |q/p|_d = 1.0010 \pm 0.0008 \quad (75.26)$$

$$\mathcal{A}_{\text{SL}}^s = -0.0006 \pm 0.0028, \text{ or } |q/p|_s = 1.0003 \pm 0.0014 \quad (75.27)$$

with a correlation coefficient of  $-0.054$  between  $\mathcal{A}_{\text{SL}}^d$  and  $\mathcal{A}_{\text{SL}}^s$ . These results show no evidence of  $CP$  violation and don't constrain yet the Standard Model.

$CP$  violation induced by  $B_s^0-\bar{B}_s^0$  mixing in  $b \rightarrow c\bar{c}s$  decays has been a field of very active study in the past decade. In addition to the previously mentioned  $B_s^0 \rightarrow J/\psi K^+ K^-$  (including  $B_s^0 \rightarrow J/\psi \phi$ ) and  $B_s^0 \rightarrow \psi(2S)\phi$  studies, the decay modes  $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$  (including  $B_s^0 \rightarrow J/\psi f_0(980)$ ) [93] and  $B_s^0 \rightarrow D_s^+ D_s^-$  [29] have also been analyzed by LHCb to measure  $\phi_s^{c\bar{c}s}$ , without the need for an angular analysis. The  $J/\psi \pi^+ \pi^-$  final state has been shown indeed to be (very close to) a pure  $CP$ -odd state [107]. A two-dimensional fit [8] of all published results [28, 29, 70–72, 74–76, 93] in the  $(\phi_s^{c\bar{c}s}, \Delta\Gamma_s)$  plane yields

$$\phi_s^{c\bar{c}s} = -0.051 \pm 0.023. \quad (75.28)$$

Adding to this fit the new ATLAS results recently submitted for publication [73] leads to the preliminary average  $\phi_s^{c\bar{c}s} = -0.057 \pm 0.021$ , with the overall situation shown on Fig. 75.3.

This experimental value is consistent with the Standard Model prediction for  $\phi_s^{c\bar{c}s}$ , which is equal to  $-2\beta_s = -2\arg(-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)) = -0.0370_{-0.0008}^{+0.0007}$  [57] assuming negligible Penguin pollution.

### 75.7 Summary

$B^0-\bar{B}^0$  mixing has been and still is a field of intense study. The mass differences in the  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  systems are known to relative precisions of 0.38% and 0.11%, respectively. The non-zero decay width difference in the  $B_s^0-\bar{B}_s^0$  system is well established, with a relative difference of  $\Delta\Gamma_s/\Gamma_s = (12.9 \pm 0.6)\%$ , meaning that the heavy state of the  $B_s^0-\bar{B}_s^0$  system lives  $\sim 14\%$  longer than the light state. In contrast, the relative decay width difference in the  $B_d^0-\bar{B}_d^0$  system,  $\Delta\Gamma_d/\Gamma_d = (0.1 \pm 1.0)\%$ , is still consistent with zero.  $CP$  violation in  $B_d^0-\bar{B}_d^0$  or  $B_s^0-\bar{B}_s^0$  mixing has not been observed yet, with precisions on the semileptonic asymmetries below 0.3%.  $CP$  violation induced by  $B_s^0-\bar{B}_s^0$  mixing in  $b \rightarrow c\bar{c}s$  transitions has not yet been observed either, with an uncertainty on the  $\phi_s^{c\bar{c}s}$  phase of 23 mrad. Despite the recent improvements, all observations remain consistent with the Standard Model expectations.

However, the measurements where New Physics might show up are still statistically limited. More results are awaited from the LHC experiments and Belle II, with promising prospects for the investigation of the  $CP$ -violating phase  $\arg(-M_{12}/\Gamma_{12})$  and an improved determination of  $\phi_s^{c\bar{c}s}$ .

Mixing studies have clearly reached the stage of precision measurements, where much effort is needed, both on the experimental and theoretical sides, in particular to further reduce the hadronic uncertainties of lattice QCD calculations. In the long term, a stringent check of the consistency of the  $B_d^0$  and  $B_s^0$  mixing amplitudes (magnitudes and phases) with all other measured flavor-physics observables will be possible within the Standard Model, leading to very tight limits on (or otherwise a long-awaited surprize about) New Physics.

### References

- [1] T. D. Lee and C. S. Wu, *Ann. Rev. Nucl. Part. Sci.* **16**, 511 (1966); I. I. Bigi and A. I. Sanda, *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.* **9**, 1 (2009), second edition, first published in 2000; G. C. Branco, L. Lavoura and J. P. Silva, *Int. Ser. Monogr. Phys.* **103**, 1 (1999).
- [2] See the review on  $CP$  violation in the quark sector by T. Gershon and Y. Nir in this publication.
- [3] A. J. Buras, W. Slominski and H. Steger, *Nucl. Phys.* **B245**, 369 (1984).
- [4] T. Inami and C. S. Lim, *Prog. Theor. Phys.* **65**, 297 (1981), erratum *ibid.* **65**, 1772 (1981); for the power-like approximation, see A. J. Buras and R. Fleischer, page 91 in “Heavy Flavours II,” eds. A. J. Buras and M. Lindner, Singapore World Scientific (1998).
- [5] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
- [6] I. I. Bigi *et al.*, in “ $CP$  violation,” ed. C. Jarlskog, Singapore World Scientific (1989).
- [7] A. Lenz and U. Nierste (2011), [arXiv:1102.4274]; A. Lenz and U. Nierste, *JHEP* **06**, 072 (2007), [hep-ph/0612167].
- [8] Y. Amhis *et al.* (HFLAV) (2019), [arXiv:1909.12524]; the combined results on  $b$ -hadron fractions, lifetimes and mixing parameters published in this *Review* have been obtained by the  $B$  oscillations working group of the Heavy Flavor Averaging (HFLAV) group, using the methods and procedures described in Chapter 4 of the above paper, after updating the list of inputs; for more information, see <https://hflav.web.cern.ch/>.

- [9] T. Jubb *et al.*, *Nucl. Phys.* **B915**, 431 (2017), [arXiv:1603.07770]; M. Artuso, G. Borissov and A. Lenz, *Rev. Mod. Phys.* **88**, 045002 (2016), [arXiv:1511.09466].
- [10] C. Albajar *et al.* (UA1), *Phys. Lett.* **B186**, 247 (1987), erratum *ibid.* **B197**, 565 (1987).
- [11] H. Albrecht *et al.* (ARGUS), *Phys. Lett.* **B192**, 245 (1987).
- [12] H. G. Moser and A. Roussarie, *Nucl. Instrum. Meth.* **A384**, 491 (1997).
- [13] SLD collab., SLAC-PUB-7228, SLAC-PUB-7229, and SLAC-PUB-7230, *28th Int. Conf. on High Energy Physics*, Warsaw (1996); J. L. Wittlin, *A measurement of the time dependence of  $B_d^0-\bar{B}_d^0$  mixing with kaon tagging*, Ph.D. thesis, Massachusetts U., Amherst (2001), URL <http://wwwlib.umi.com/dissertations/fullcit?p3027272>.
- [14] ALEPH collab., contrib. 596 to *Int. Europhysics Conf. on High Energy Physics*, Jerusalem (1997).
- [15] K. Abe *et al.* (SLD), *Phys. Rev.* **D67**, 012006 (2003), [hep-ex/0209002].
- [16] J. Abdallah *et al.* (DELPHI), *Eur. Phys. J.* **C28**, 155 (2003), [hep-ex/0303032].
- [17] F. Abe *et al.* (CDF), *Phys. Rev. Lett.* **80**, 2057 (1998), [hep-ex/9712004]; F. Abe *et al.* (CDF), *Phys. Rev.* **D59**, 032001 (1999), [hep-ex/9806026]; F. Abe *et al.* (CDF), *Phys. Rev.* **D60**, 051101 (1999); F. Abe *et al.* (CDF), *Phys. Rev.* **D60**, 072003 (1999), [hep-ex/9903011]; T. Affolder *et al.* (CDF), *Phys. Rev.* **D60**, 112004 (1999), [hep-ex/9907053].
- [18] R. Barate *et al.* (ALEPH), *Eur. Phys. J.* **C4**, 367 (1998); R. Barate *et al.* (ALEPH), *Eur. Phys. J.* **C7**, 553 (1999), [hep-ex/9811018].
- [19] R. Aaij *et al.* (LHCb), *JINST* **10**, P10005 (2015), [arXiv:1507.07892].
- [20] P. Abreu *et al.* (DELPHI), *Eur. Phys. J.* **C16**, 555 (2000), [hep-ex/0107077].
- [21] K. Anikeev *et al.* (2002), [hep-ph/0201071].
- [22] A. Abulencia *et al.* (CDF), *Phys. Rev. Lett.* **97**, 242003 (2006), [hep-ex/0609040].
- [23] V. M. Abazov *et al.* (DØ), *Phys. Rev.* **D74**, 112002 (2006), [hep-ex/0609034].
- [24] V. M. Abazov *et al.* (DØ), *Phys. Rev. Lett.* **101**, 241801 (2008), [arXiv:0802.2255].
- [25] R. Aaij *et al.* (LHCb), *Eur. Phys. J.* **C72**, 2022 (2012), [arXiv:1202.4979].
- [26] R. Aaij *et al.* (LHCb), *JINST* **11**, P05010 (2016), [arXiv:1602.07252].
- [27] R. Aaij *et al.* (LHCb), *Eur. Phys. J.* **C77**, 238 (2017), [arXiv:1610.06019].
- [28] R. Aaij *et al.* (LHCb), *Phys. Rev. Lett.* **114**, 041801 (2015), [arXiv:1411.3104]; R. Aaij *et al.* (LHCb), *Eur. Phys. J.* **C79**, 706 (2019), erratum to appear, [arXiv:1906.08356].
- [29] R. Aaij *et al.* (LHCb), *Phys. Rev. Lett.* **113**, 211801 (2014), [arXiv:1409.4619].
- [30] R. Aaij *et al.* (LHCb), *Phys. Rev. Lett.* **117**, 261801 (2016), [arXiv:1608.06620].
- [31] B. Aubert *et al.* (BaBar), *Phys. Rev. Lett.* **94**, 161803 (2005), [hep-ex/0408127]; K. F. Chen *et al.* (Belle), *Phys. Rev.* **D72**, 012004 (2005), [hep-ex/0504023].
- [32] Throughout this document we omit references of results that have been replaced by new published measurements.
- [33] D. Buskulic *et al.* (ALEPH), *Z. Phys.* **C75**, 397 (1997).
- [34] P. Abreu *et al.* (DELPHI), *Z. Phys.* **C76**, 579 (1997).
- [35] M. Acciarri *et al.* (L3), *Eur. Phys. J.* **C5**, 195 (1998).
- [36] G. Alexander *et al.* (OPAL), *Z. Phys.* **C72**, 377 (1996); K. Ackerstaff *et al.* (OPAL), *Z. Phys.* **C76**, 417 (1997), [hep-ex/9707010]; G. Abbiendi *et al.* (OPAL), *Phys. Lett.* **B493**, 266 (2000), [hep-ex/0010013].

- [37] K. Ackerstaff *et al.* (OPAL), *Z. Phys.* **C76**, 401 (1997), [[hep-ex/9707009](#)].
- [38] B. Aubert *et al.* (BaBar), *Phys. Rev. Lett.* **88**, 221802 (2002), [[hep-ex/0112044](#)]; B. Aubert *et al.* (BaBar), *Phys. Rev.* **D66**, 032003 (2002), [[hep-ex/0201020](#)]; B. Aubert *et al.* (BaBar), *Phys. Rev. Lett.* **88**, 221803 (2002), [[hep-ex/0112045](#)]; B. Aubert *et al.* (BaBar), *Phys. Rev.* **D67**, 072002 (2003), [[hep-ex/0212017](#)]; B. Aubert *et al.* (BaBar), *Phys. Rev.* **D73**, 012004 (2006), [[hep-ex/0507054](#)].
- [39] N. C. Hastings *et al.* (Belle), *Phys. Rev.* **D67**, 052004 (2003), [[hep-ex/0212033](#)]; Y. Zheng *et al.* (Belle), *Phys. Rev.* **D67**, 092004 (2003), [[hep-ex/0211065](#)]; K. Abe *et al.* (Belle), *Phys. Rev.* **D71**, 072003 (2005), erratum *ibid.* **D71**, 079903 (2005), [[hep-ex/0408111](#)].
- [40] R. Aaij *et al.* (LHCb), *Phys. Lett.* **B709**, 177 (2012), [[arXiv:1112.4311](#)].
- [41] R. Aaij *et al.* (LHCb), *Phys. Lett.* **B719**, 318 (2013), [[arXiv:1210.6750](#)].
- [42] R. Aaij *et al.* (LHCb), *Eur. Phys. J.* **C73**, 2655 (2013), [[arXiv:1308.1302](#)].
- [43] P. Huang *et al.*, *Eur. Phys. J.* **C76**, 422 (2016), [[arXiv:1505.03672](#)].
- [44] H. Albrecht *et al.* (ARGUS), *Z. Phys.* **C55**, 357 (1992); H. Albrecht *et al.* (ARGUS), *Phys. Lett.* **B324**, 249 (1994).
- [45] J. E. Bartelt *et al.* (CLEO), *Phys. Rev. Lett.* **71**, 1680 (1993).
- [46] B. H. Behrens *et al.* (CLEO), *Phys. Lett.* **B490**, 36 (2000), [[hep-ex/0005013](#)].
- [47] B. Aubert *et al.* (BaBar), *Phys. Rev. Lett.* **92**, 181801 (2004), [[hep-ex/0311037](#)]; B. Aubert *et al.* (BaBar), *Phys. Rev.* **D70**, 012007 (2004), [[hep-ex/0403002](#)].
- [48] T. Higuchi *et al.*, *Phys. Rev.* **D85**, 071105 (2012), [[arXiv:1203.0930](#)].
- [49] R. Aaij *et al.* (LHCb), *JHEP* **04**, 114 (2014), [[arXiv:1402.2554](#)].
- [50] M. Aaboud *et al.* (ATLAS), *JHEP* **06**, 081 (2016), [[arXiv:1605.07485](#)].
- [51] A. M. Sirunyan *et al.* (CMS), *Eur. Phys. J.* **C78**, 457 (2018), erratum *ibid.* **C78**, 561 (2018), [[arXiv:1710.08949](#)].
- [52] See the review on the CKM quark-mixing matrix by A. Ceccucci, Z. Ligeti, and Y. Sakai in this publication.
- [53] S. Aoki *et al.* (Flavour Lattice Averaging Group) (2019), [[arXiv:1902.08191](#)].
- [54] R. Aaij *et al.* (LHCb), *New J. Phys.* **15**, 053021 (2013), [[arXiv:1304.4741](#)].
- [55] A. Bazavov *et al.* (Fermilab Lattice and MILC), *Phys. Rev.* **D93**, 113016 (2016), [[arXiv:1602.03560](#)].
- [56] M. Bona *et al.* (UTfit), *JHEP* **10**, 081 (2006), [[hep-ph/0606167](#)]; updated results at <http://www.utfit.org/>.
- [57] J. Charles *et al.*, *Phys. Rev.* **D91**, 073007 (2015), [[arXiv:1501.05013](#)]; updated results at <http://ckmfitter.in2p3.fr/>.
- [58] K. Hartkorn and H. G. Moser, *Eur. Phys. J.* **C8**, 381 (1999).
- [59] M. Acciarri *et al.* (L3), *Phys. Lett.* **B438**, 417 (1998).
- [60] D. Buskulic *et al.* (ALEPH), *Phys. Lett.* **B377**, 205 (1996); K. Ackerstaff *et al.* (OPAL), *Phys. Lett.* **B426**, 161 (1998), [[hep-ex/9802002](#)]; F. Abe *et al.* (CDF), *Phys. Rev.* **D59**, 032004 (1999), [[hep-ex/9808003](#)]; V. M. Abazov *et al.* (DØ), *Phys. Rev. Lett.* **114**, 062001 (2015), [[arXiv:1410.1568](#)]; T. Aaltonen *et al.* (CDF), *Phys. Rev. Lett.* **107**, 272001 (2011), [[arXiv:1103.1864](#)]; R. Aaij *et al.* (LHCb), *Phys. Rev. Lett.* **113**, 172001 (2014), [[arXiv:1407.5873](#)]; R. Aaij *et al.* (LHCb), *Phys. Rev. Lett.* **119**, 101801 (2017), [[arXiv:1705.03475](#)].

- [61] R. Aaij *et al.* (LHCb), *Phys. Rev. Lett.* **112**, 111802 (2014), [arXiv:1312.1217].
- [62] R. Aaij *et al.* (LHCb), *Phys. Lett.* **B736**, 446 (2014), [arXiv:1406.7204].
- [63] R. Aaij *et al.* (LHCb), *Phys. Lett.* **B707**, 349 (2012), [arXiv:1111.0521].
- [64] R. Aaij *et al.* (LHCb), *Phys. Lett.* **B762**, 484 (2016), [arXiv:1607.06314].
- [65] T. Aaltonen *et al.* (CDF), *Phys. Rev.* **D84**, 052012 (2011), [arXiv:1106.3682]; V. M. Abazov *et al.* (DØ), *Phys. Rev.* **D94**, 012001 (2016), [arXiv:1603.01302].
- [66] R. Aaij *et al.* (LHCb), *Phys. Rev.* **D87**, 112010 (2013), [arXiv:1304.2600].
- [67] R. Aaij *et al.* (LHCb), *Nucl. Phys.* **B873**, 275 (2013), [arXiv:1304.4500].
- [68] R. Aaij *et al.* (LHCb), *Phys. Rev. Lett.* **118**, 191801 (2017), [arXiv:1703.05747].
- [69] R. Fleischer and R. Knegjens, *Eur. Phys. J.* **C71**, 1789 (2011), [arXiv:1109.5115].
- [70] T. Aaltonen *et al.* (CDF), *Phys. Rev. Lett.* **109**, 171802 (2012), [arXiv:1208.2967].
- [71] V. M. Abazov *et al.* (DØ), *Phys. Rev.* **D85**, 032006 (2012), [arXiv:1109.3166].
- [72] G. Aad *et al.* (ATLAS), *Phys. Rev.* **D90**, 052007 (2014), [arXiv:1407.1796]; G. Aad *et al.* (ATLAS), *JHEP* **08**, 147 (2016), [arXiv:1601.03297].
- [73] G. Aad *et al.* (ATLAS) (2020), [arXiv:2001.07115].
- [74] V. Khachatryan *et al.* (CMS), *Phys. Lett.* **B757**, 97 (2016), [arXiv:1507.07527].
- [75] R. Aaij *et al.* (LHCb), *JHEP* **08**, 037 (2017), [arXiv:1704.08217].
- [76] R. Aaij *et al.* (LHCb), *Phys. Lett.* **B762**, 253 (2016), [arXiv:1608.04855].
- [77] R. Aaij *et al.* (LHCb), *Phys. Rev. Lett.* **108**, 241801 (2012), [arXiv:1202.4717].
- [78] S. Schael *et al.* (ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group), *Phys. Rept.* **427**, 257 (2006), [hep-ex/0509008]; we use the  $\bar{\chi}$  average given in Eq. (5.39).
- [79] D. Acosta *et al.* (CDF), *Phys. Rev.* **D69**, 012002 (2004), [hep-ex/0309030].
- [80] V. M. Abazov *et al.* (DØ), *Phys. Rev.* **D74**, 092001 (2006), [hep-ex/0609014].
- [81] P. Abreu *et al.* (DELPHI), *Phys. Lett.* **B289**, 199 (1992); P. D. Acton *et al.* (OPAL), *Phys. Lett.* **B295**, 357 (1992); D. Buskulic *et al.* (ALEPH), *Phys. Lett.* **B361**, 221 (1995).
- [82] P. Abreu *et al.* (DELPHI), *Z. Phys.* **C68**, 375 (1995); R. Barate *et al.* (ALEPH), *Eur. Phys. J.* **C2**, 197 (1998).
- [83] D. Buskulic *et al.* (ALEPH), *Phys. Lett.* **B384**, 449 (1996); J. Abdallah *et al.* (DELPHI), *Eur. Phys. J.* **C44**, 299 (2005), [hep-ex/0510023].
- [84] R. Barate *et al.* (ALEPH), *Eur. Phys. J.* **C5**, 205 (1998).
- [85] J. Abdallah *et al.* (DELPHI), *Phys. Lett.* **B576**, 29 (2003), [hep-ex/0311005].
- [86] F. Abe *et al.* (CDF), *Phys. Rev.* **D60**, 092005 (1999).
- [87] T. Aaltonen *et al.* (CDF), *Phys. Rev.* **D77**, 072003 (2008), [arXiv:0801.4375]; T. Affolder *et al.* (CDF), *Phys. Rev. Lett.* **84**, 1663 (2000), [hep-ex/9909011]; the measurement of  $f_{\text{baryon}}/f_d$  in the latter paper has been updated based on; T. Aaltonen *et al.* (CDF), *Phys. Rev.* **D79**, 032001 (2009), [arXiv:0810.3213].
- [88] V. M. Abazov *et al.* (DØ), *Phys. Rev. Lett.* **99**, 052001 (2007), [arXiv:0706.1690]; V. M. Abazov *et al.* (DØ), *Phys. Rev. Lett.* **101**, 232002 (2008), [arXiv:0808.4142]; T. Aaltonen *et al.* (CDF), *Phys. Rev.* **D80**, 072003 (2009), [arXiv:0905.3123].
- [89] R. Aaij *et al.* (LHCb), *JHEP* **04**, 001 (2013), [arXiv:1301.5286]; the LHCb average of  $f_s/f_d$  has been updated in LHCb collab., LHCb-CONF-2013-011 (2013).

- [90] R. Aaij *et al.* (LHCb), *Phys. Rev.* **D85**, 032008 (2012), [arXiv:1111.2357]; R. Aaij *et al.* (LHCb), *Phys. Rev.* **D100**, 031102 (2019), [arXiv:1902.06794].
- [91] G. Aad *et al.* (ATLAS), *Phys. Rev. Lett.* **115**, 262001 (2015), [arXiv:1507.08925].
- [92] R. Aaij *et al.* (LHCb), *Phys. Rev.* **D100**, 112006 (2019), [arXiv:1910.13404].
- [93] R. Aaij *et al.* (LHCb), *Phys. Lett.* **B736**, 186 (2014), [arXiv:1405.4140]; R. Aaij *et al.* (LHCb), *Phys. Lett.* **B797**, 134789 (2019), [arXiv:1903.05530].
- [94] D. E. Jaffe *et al.* (CLEO), *Phys. Rev. Lett.* **86**, 5000 (2001), [hep-ex/0101006].
- [95] J. P. Lees *et al.* (BaBar), *Phys. Rev. Lett.* **114**, 081801 (2015), [arXiv:1411.1842].
- [96] F. Abe *et al.* (CDF), *Phys. Rev.* **D55**, 2546 (1997).
- [97] V. M. Abazov *et al.* (DØ), *Phys. Rev.* **D89**, 012002 (2014), [arXiv:1310.0447].
- [98] V. M. Abazov *et al.* (DØ), *Phys. Rev.* **D86**, 072009 (2012), [arXiv:1208.5813].
- [99] V. M. Abazov *et al.* (DØ), *Phys. Rev. Lett.* **110**, 011801 (2013), [arXiv:1207.1769].
- [100] R. Aaij *et al.* (LHCb), *Phys. Rev. Lett.* **117**, 061803 (2016), erratum *ibid.* **118**, 129903 (2017), [arXiv:1605.09768].
- [101] R. Barate *et al.* (ALEPH), *Eur. Phys. J.* **C20**, 431 (2001).
- [102] J. P. Lees *et al.* (BaBar), *Phys. Rev. Lett.* **111**, 101802 (2013), erratum *ibid.* **111**, 159901 (2013), [arXiv:1305.1575].
- [103] E. Nakano *et al.* (Belle), *Phys. Rev.* **D73**, 112002 (2006), [hep-ex/0505017].
- [104] G. Abbiendi *et al.* (OPAL), *Eur. Phys. J.* **C12**, 609 (2000), [hep-ex/9901017].
- [105] M. Beneke, G. Buchalla and I. Dunietz, *Phys. Lett.* **B393**, 132 (1997), [hep-ph/9609357]; I. Dunietz, *Eur. Phys. J.* **C7**, 197 (1999), [hep-ph/9806521].
- [106] R. Aaij *et al.* (LHCb), *Phys. Rev. Lett.* **114**, 041601 (2015), [arXiv:1409.8586].
- [107] R. Aaij *et al.* (LHCb), *Phys. Rev.* **D86**, 052006 (2012), [arXiv:1204.5643].