# 74. Polarization in $B$ Decays

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We review the notation used in polarization measurements in particle production and decay, with a particular emphasis on the $B$ decays and the *CP*-violating observables in polarization measurements. We look at several examples of vector-vector and vector-tensor $B$ meson decays, while more details about the theory and experimental results in $B$ decays can be found in a separate mini-review [1] in this *Review*.

Figure 74.1 illustrates angular observables in an example of the sequential process $ab \to X \to P_1P_2 \to (p_{11}p_{12})(p_{21}p_{22})$ [2]. The angular distributions are of particular interest because they are sensitive to spin correlations and reveal properties of particles and their interactions, such as

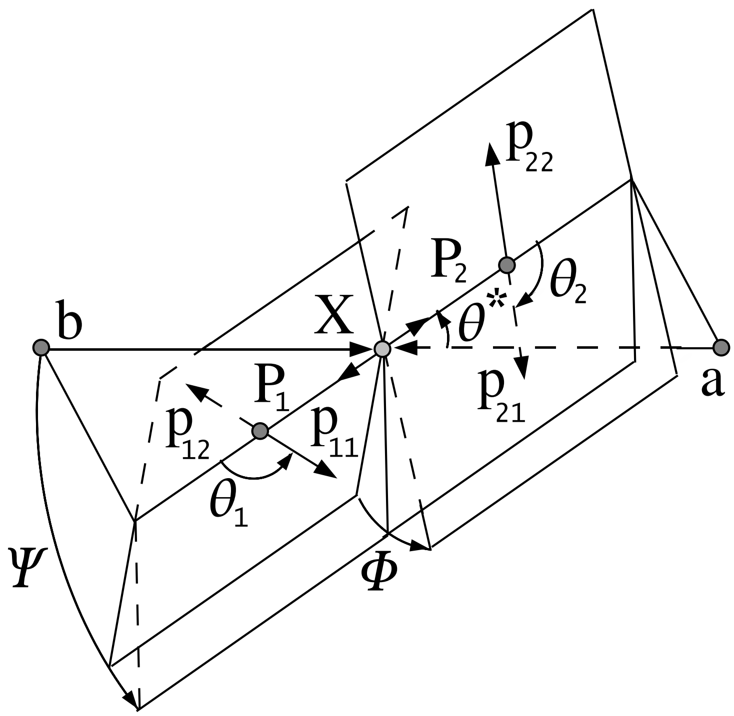


**Figure 74.1:** Definition of the production and helicity angles in the sequential process $ab \to X \to P_1P_2 \to (p_{11}p_{12})(p_{21}p_{22})$. The three helicity angles include $\theta_1$ and $\theta_2$, defined in the rest frame of the two daughters $P_1$ and $P_2$, and $\Phi$, defined in the $X$ frame as the angle between the two decay planes. The two production angles $\theta^*$ and $\Psi$ are defined in the $X$ frame, where $\Psi$ is the angle between the production plane and the average of the two decay planes.

quantum numbers and couplings. In the case of a spin-zero particle $X$, such as $B$ meson or a Higgs boson, there are no spin correlations in the production mechanism and the decay chain is to be analyzed. The angular distribution of decay products can be expressed as a function of three helicity angles which describe the alignment of the particles in the decay chain. The analyzer of the $B$-daughter polarization is normally chosen for two-body decays, as the direction of the daughters in the center-of-mass of the parent (*e.g.*, $\rho \to 2\pi$) [3], and for three-body decays as the normal to the decay plane (*e.g.*, $\omega \to 3\pi$) [4]. An equivalent set of transversity angles is sometimes used in polarization analyses [5]. The differential decay width depends on complex amplitudes $A_{\lambda_1\lambda_2}$, corresponding to the $X$-daughter helicity states $\lambda_i$.

In the case of a spin-zero $B$-meson decay, its daughter helicities are constrained to $\lambda_1 = \lambda_2 = \lambda$. Therefore we simplify amplitude notation as $A_\lambda$. Moreover, most $B$-decay polarization analyses are limited to the case when the spin of one of the $B$-meson daughters is 1. In that case, there are only three independent amplitudes corresponding to $\lambda = 0$ or $\pm 1$ [6], where the last two can be expressed in terms of parity-even and parity-odd amplitudes $A_{\parallel,\perp} = (A_{+1} \pm A_{-1})/\sqrt{2}$. The overall decay amplitude involves three complex terms proportional to the above amplitudes and the Wigner $d$ functions of helicity angles. The exact angular dependence would depend on the quantum numbers of the $B$-meson daughters and of their decay products, and can be found in the literature [6, 7]. When both $B$-meson daughters are tensor mesons and the smaller of the two daughter spins is $J_1 > 1$, this formalism can be easily extended by introducing the parity-even and parity-odd amplitudes of higher order $A_{\parallel n, \perp n} = (A_{+n} \pm A_{-n})/\sqrt{2}$, with $1 < n \leq J_1$, while the general angular parameterization may be found in Ref. [7]. However, we limit the following discussion to $J_1 = 1$. The differential decay rate would involve six real quantities $\alpha_i$, including interference terms,

$$\frac{d\mathit{\Gamma}}{\mathit{\Gamma}\, d\cos\theta_1\, d\cos\theta_2\, d\mathit{\Phi}} = \sum_i \alpha_i\; f_i\left(\cos\theta_1,\, \cos\theta_2,\, \mathit{\Phi}\right), \tag{74.1}$$

where each $f_i\left(\cos\theta_1,\, \cos\theta_2,\, \mathit{\Phi}\right)$ has unique angular dependence specific to particle quantum numbers, and the $\alpha_i$ parameters are defined as:

$$\alpha_1 = \frac{|A_0|^2}{\Sigma|A_\lambda|^2} = f_L\,, \tag{74.2}$$

$$\alpha_2 = \frac{|A_\parallel|^2 + |A_\perp|^2}{\Sigma|A_\lambda|^2} = (1 - f_L)\,, \tag{74.3}$$

$$\alpha_3 = \frac{|A_\parallel|^2 - |A_\perp|^2}{\Sigma|A_\lambda|^2} = (1 - f_L - 2\, f_\perp)\,, \tag{74.4}$$

$$\alpha_4 = \frac{\Im m(A_\perp A_\parallel^*)}{\Sigma|A_\lambda|^2} = \sqrt{f_\perp(1 - f_L - f_\perp)}\, \sin(\phi_\perp - \phi_\parallel)\,, \tag{74.5}$$

$$\alpha_5 = \frac{\Re e(A_\parallel A_0^*)}{\Sigma|A_\lambda|^2} = \sqrt{f_L\,(1 - f_L - f_\perp)}\, \cos(\phi_\parallel)\,, \tag{74.6}$$

$$\alpha_6 = \frac{\Im m(A_\perp A_0^*)}{\Sigma|A_\lambda|^2} = \sqrt{f_\perp\, f_L}\, \sin(\phi_\perp)\,, \tag{74.7}$$

where the amplitudes have been expressed with the help of polarization parameters $f_L$, $f_\perp$, $\phi_\parallel$, and $\phi_\perp$ defined in Table 74.1. Note that the terms proportional to $\Re e(A_\perp A_\parallel^*)$, $\Im m(A_\parallel A_0^*)$, and $\Re e(A_\perp A_0^*)$ are absent in Eqs. (74.2-74.7). However, these terms may appear for some three-body decays of a $B$-meson daughter, see Ref. [7].

Overall, six real parameters describe three complex amplitudes $A_0$, $A_\parallel$, and $A_\perp$. These could be chosen to be the four polarization parameters $f_L$, $f_\perp$, $\phi_\parallel$, and $\phi_\perp$, one overall size normalization, such as decay rate $\mathit{\Gamma}$, or branching fraction $\mathcal{B}$, and one overall phase $\delta_0$. The phase convention is arbitrary for an isolated $B$ decay mode. However, for several $B$ decays, the relative phase could produce meaningful and observable effects through interference with other $B$ decays with the same final states, such as for $B \to V K_J^*$ with $J = 0, 1, 2, 3, 4, ...$ The phase could be referenced to the single $B \to V K_0^*$ amplitude $A_{00}$ in such a case, as shown in Table 74.1. Here $V$ stands for any spin-one vector meson.

Moreover, *CP* violation can be tested in the angular distribution of the decay as the difference between the $B$ and $\overline{B}$. Each of the six real parameters describing the three complex amplitudes

**Table 74.1:** Rate, polarization, and *CP*-asymmetry parameters defined for the $B$-meson decays to mesons with non-zero spin. Numerical examples are shown for the average of the $B^0 \to \varphi K^*(892)^0$ decay measurements obtained from BABAR [8], Belle [9], and LHCb [10]. The first six parameters are defined under the assumption of no *CP* violation in decay, while they are averaged between the $\overline{B}$ and $B$ parameters in general. The last six parameters involve differences between the $\overline{B}$ and $B$ meson decay parameters. The phase convention $\delta_0$ is chosen with respect to a single $A_{00}$ amplitude from a reference $B$ decay mode, which is $B^0 \to \varphi K_0^*(1430)^0$ for numerical results.

| parameter | definition | average |
|---|---|---|
| $\mathcal{B}$ | $\Gamma/\Gamma_{\rm total}$ | $(10.1^{+0.6}_{-0.5}) \times 10^{-6}$ |
| $f_L$ | $\lvert A_0\rvert^2/\Sigma\lvert A_\lambda\rvert^2$ | $0.497 \pm 0.017$ |
| $f_\perp$ | $\lvert A_\perp\rvert^2/\Sigma\lvert A_\lambda\rvert^2$ | $0.225 \pm 0.015$ |
| $\phi_\parallel - \pi$ | $\arg(A_\parallel/A_0) - \pi$ | $-0.712 \pm 0.058$ |
| $\phi_\perp - \pi$ | $\arg(A_\perp/A_0) - \pi$ | $-0.615 \pm 0.056$ |
| $\delta_0 - \pi$ | $\arg(A_{00}/A_0) - \pi$ | $-0.26 \pm 0.10$ |
| $\mathcal{A}_{CP}$ | $(\bar{\Gamma} - \Gamma)/(\bar{\Gamma} + \Gamma)$ | $-0.003 \pm 0.038$ |
| $\mathcal{A}^0_{CP}$ | $(\bar{f}_L - f_L)/(\bar{f}_L + f_L)$ | $-0.007 \pm 0.030$ |
| $\mathcal{A}^\perp_{CP}$ | $(\bar{f}_\perp - f_\perp)/(\bar{f}_\perp + f_\perp)$ | $-0.014 \pm 0.057$ |
| $\Delta\phi_\parallel$ | $(\bar{\phi}_\parallel - \phi_\parallel)/2$ | $+0.051 \pm 0.053$ |
| $\Delta\phi_\perp$ | $(\bar{\phi}_\perp - \phi_\perp - \pi)/2$ | $+0.075 \pm 0.050$ |
| $\Delta\delta_0$ | $(\bar{\delta}_0 - \delta_0)/2$ | $+0.13 \pm 0.08$ |

would have a counterpart *CP*-asymmetry term, corresponding to three direct-*CP* asymmetries in three amplitudes, and three *CP*-violating phase differences, equivalent to the phase measurements from the mixing-induced *CP* asymmetries in the time evolution of $B$-decays [1]. In Table 74.1 and Ref. [11], these are chosen to be the direct-*CP* asymmetries in the overall decay rate $\mathcal{A}_{CP}$, in the $f_L$ fraction $\mathcal{A}^0_{CP}$, and in the $f_\perp$ fraction $\mathcal{A}^\perp_{CP}$, and three weak phase differences:

$$\Delta\phi_\parallel = \frac{1}{2}\arg(\bar{A}_\parallel A_0/A_\parallel \bar{A}_0)\,, \tag{74.8}$$

$$\Delta\phi_\perp = \frac{1}{2}\arg(\bar{A}_\perp A_0/A_\perp \bar{A}_0) - \frac{\pi}{2}\,, \tag{74.9}$$

$$\Delta\delta_0 = \frac{1}{2}\arg(\bar{A}_{00} A_0/A_{00} \bar{A}_0)\,. \tag{74.10}$$

The $\frac{\pi}{2}$ term in Eq. (74.8) reflects the fact that $A_\perp$ and $\bar{A}_\perp$ differ in phase by $\pi$ if *CP* is conserved. The two parameters $\Delta\phi_\parallel$ and $\Delta\phi_\perp$ are equivalent to triple-product asymmetries constructed from the vectors describing the decay angular distribution [12]. The *CP*-violating phase difference in the reference decay mode [11] is, in the Wolfenstein CKM quark-mixing phase convention,

$$\Delta\phi_{00} = \frac{1}{2}\arg(A_{00}/\bar{A}_{00})\,. \tag{74.11}$$

This can be measured only together with the mixing-induced phase difference for some of the neutral $B$-meson decays similar to other mixing-induced *CP* asymmetry measurements [1].

It may not always be possible to have a phase-reference decay mode which would define $\delta_0$ and $\Delta\delta_0$ parameters. In that case, it may be possible to define the phase difference directly similarly to Eq. (74.11):

$$\Delta\phi_0 = \frac{1}{2}\arg(A_0/\bar{A}_0)\,. \tag{74.12}$$

One can measure the angles of the CKM unitarity triangle, assuming Standard Model contributions to the $\Delta\phi_0$ and $B$-mixing phases. Examples include measurements of $\beta = \phi_1$ with $B \to J/\psi K^*$ and $\alpha = \phi_2$ with $B \to \rho\rho$.

Most of the $B$ decays that arise from tree-level $b \to c$ transitions have the amplitude hierarchy $|A_0| > |A_+| > |A_-|$ which is expected from analyses based on quark-helicity conservation [13]. The larger the mass of the vector-meson daughters, the weaker the inequality. The $B$ meson decays to heavy vector particles with charm, such as $B \to J/\psi K^*$, $\psi(2S)K^*$, $\chi_{c1}K^*$, $D^*\rho$, $D^*K^*$, $D^*D^*$, and $D^*D_s^*$, show a substantial fraction of the amplitudes corresponding to transverse polarization of the vector mesons ($A_{\pm1}$), in agreement with the factorization prediction. The detailed amplitude analysis of the $B \to J/\psi K^*$ decays has been performed by the BABAR [14], Belle [15], CDF [16], CLEO [17], D0 [18], and LHCb [19] collaborations. Most analyses are performed under the assumption of the absence of direct *CP* violation. The parameter values are given in the particle listing of this *Review*. The difference between the strong phases $\phi_\parallel$ and $\phi_\perp$ deviates significantly from zero. The measurements [14,15] of *CP*-violating terms similar to those in $B \to \varphi K^*$ [8] shown in Table 74.1 are consistent with zero.

In addition, the mixing-induced *CP*-violating asymmetry is measured in the $B^0 \to J/\psi K^{*0}$ decay [1,14,15] where angular analysis allows one to separate *CP*-eigenstate amplitudes. This allows one to resolve the sign ambiguity of the $\cos 2\beta$ ($\cos 2\phi_1$) term that appears in the time-dependent angular distribution due to interference of parity-even and parity-odd terms. This analysis relies on the knowledge of discrete ambiguities in the strong phases $\phi_\parallel$ and $\phi_\perp$, as discussed below. The BABAR experiment used a method based on the dependence on the $K\pi$ invariant mass of the interference between the $S$- and $P$-waves to resolve the discrete ambiguity in the determination of the strong phases $(\phi_\parallel, \phi_\perp)$ in $B \to J/\psi K^*$ decays [14]. The result is in agreement with the amplitude hierarchy expectation [13]. The CDF [20], D0 [21], and LHCb [22] experiments have studied the $B_s^0 \to J/\psi(K^+K^-)$, $J/\psi(\pi^+\pi^-)$, $\psi(K^+\pi^-)$ decays and provided the lifetime, polarization, and phase measurements.

The amplitude hierarchy $|A_0| \gg |A_+| \gg |A_-|$ was expected in $B$ decays to light vector particles in both penguin transitions [23, 24] and tree-level transitions [13]. There is confirmation by the BABAR and Belle experiments of predominantly longitudinal polarization in the tree-level $b \to u$ transition, such as $B^0 \to \rho^+\rho^-$ [25], $B^+ \to \rho^0\rho^+$ [26], and $B^+ \to \omega\rho^+$ [27]; this is consistent with the analysis of the quark helicity conservation [13]. Because the longitudinal amplitude dominates the decay, a detailed amplitude analysis is not possible with current $B$ samples, and limits on the transverse amplitude fraction are obtained. The small branching fractions of $B^0 \to \rho^0\rho^0, \omega\rho^0, \omega\omega$ [27–30] indicate that $b \to d$ penguin pollution is small in the charmless, strangeless vector-vector $B$ decays. There is a measurement of large longitudinal polarization in $B^0 \to \rho^0\rho^0$ [28–30] decays. The fraction of transverse polarization is large in decays to heavier mesons such as $B^0 \to a_1(1260)^+a_1(1260)^-$ [31].

The interest in the polarization and *CP*-asymmetry measurements in penguin transition, such as $b \to s$ decays $B \to \varphi K^*$, $\rho K^*$, $\omega K^*$, or $B_s^0 \to \varphi\varphi$, $K^*K^*$, and $b \to d$ decay $B \to K^*\bar{K}^*$, is motivated by their potential sensitivity to physics beyond the Standard Model. The decay amplitudes for $B \to \varphi K^*$ have been measured by the BABAR, Belle, and LHCb experiments [9–11, 32, 33]. The fractions of longitudinal polarization are $f_L = 0.50 \pm 0.05$ for the $B^+ \to \varphi K^{*+}$ decay and $f_L = 0.497 \pm 0.017$ for the $B^0 \to \varphi K^{*0}$ decay. These indicate significant departure from the naive expectation of predominant longitudinal polarization, suggesting other contributions to the decay amplitude, previously neglected, either within the Standard Model, such as penguin annihilation [34] or QCD rescattering [35], or from physics beyond the Standard Model [36]. The complete set of twelve amplitude parameters measured in the $B^0 \to \varphi K^{*0}$ decay is given in Table 74.1. Several other parameters could be constructed from the above twelve parameters, as suggested in Ref. [37].

The discrete ambiguity in the phase $(\phi_\parallel, \phi_\perp, \Delta\phi_\parallel, \Delta\phi_\perp)$ measurements has been resolved by BABAR in favor of $|A_+| \gg |A_-|$ through interference between the $S$- and $P$-waves of $K\pi$. The search for vector-tensor and vector-axialvector $B \to \varphi K_J^{(*)}$ decays with $J = 1, 2, 3, 4$ revealed a large fraction of longitudinal polarization in the decay $B \to \varphi K_2^*(1430)$ with $f_L = 0.90^{+0.06}_{-0.07}$ [11, 38], but large contribution of transverse amplitude in $B \to \varphi K_1(1270)$ with $f_L = 0.46^{+0.13}_{-0.15}$ [39].

Like $B \to \varphi K^*$, the decays $B \to \rho K^*$ and $B \to \omega K^*$ may be sensitive to New Physics. Measurements of the longitudinal polarization fraction in $B \to \rho K^*$ [40] and in both vector-vector and vector-tensor final states of $B \to \omega K_J^*$ [27] by BABAR and Belle reveal a large fraction of transverse polarization, indicating an anomaly similar to $B \to \varphi K^*$ except for a different pattern in vector-tensor final states. An angular analysis of the $B^0 \to \rho^0 K^{*0}$ decay mode by LHCb [41] provides much higher precision and indicates remarkably small longitudinal polarization fraction and a significant direct $CP$ asymmetry observed in angular distributions of $B \to VV$ decays for the first time. A large transverse polarization is also observed in the $B_s^0 \to \varphi\varphi$ decay by CDF [42] and LHCb [43], $B_s^0 \to K^{*0}\bar{K}^{*0}$ decays by LHCb [44], and $B_s^0 \to \varphi K^{*0}$ decays by LHCb [45]. At the same time, measurement of the polarization in the $b \to d$ penguin decays $B \to K^*\bar{K}^*$ indicates a large fraction of longitudinal polarization [44, 46]. The LHCb experiment has also provided the very first polarization results on the tensor-tensor, as well as vector-tensor, decays of the $B_s^0$ meson in the $(K\pi)(K\pi)$ final state [44]. The polarization pattern in penguin-dominated $B$-meson decays is not fully understood [34–36].

The three-body semileptonic $B$-meson decays, such as $B \to V\ell_1\ell_2$, share many features with the two-body $B \to VV$ decays. Their differential decay width can be parameterized with the two helicity angles defined in the $V$ and $(\ell_1\ell_2)$ frames and with the azimuthal angle, as defined in Fig. 74.1. However, since the $(\ell_1\ell_2)$ pair does not come from an on-shell particle, the angular distribution is unique to each point in the dilepton mass $m_{\ell\ell}$ spectrum. The polarization measurements as a function of $m_{\ell\ell}$ provide complementary information on physics beyond the Standard Model, as discussed for $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays in Ref. [47]. The data in these modes have been analyzed by the BABAR, Belle, CDF, CMS, and LHCb experiments [48–53].

The examples of the angular distributions and observables in $B \to K^*\ell^+\ell^-$ are discussed in Ref. [47]. Two angular observables have been measured in this decay in certain ranges of the dilepton mass $m_{\ell\ell}$. One parameter is the fraction of longitudinal polarization $F_L$, which is determined by the $K^*$ angular distribution and is similar to $f_L$ defined for exclusive two-body decays. The other parameter is the forward-backward asymmetry of the lepton pair $A_{FB}$, which is the asymmetry of the decay rate with positive and negative values of $\cos\theta_1$. A complete set of observables and angular terms has been adopted by the LHCb collaboration [52] following Ref. [47] with the $F_L$, $A_{FB}$, and $S_3 - S_9$ coefficients in the angular distributions. Additional set of optimized observables $P_i^{(\prime)}$ is derived from those, for example $P_2 = 2A_{FB}/(3 - 3F_L)$ and $P_5' = S_5/\sqrt{F_L(1 - F_L)}$. These observables have the advantage that the leading form-factor uncertainties cancel. There have been hints of deviations from SM in the measurement of $P_5'$ and lepton flavor universality [48–53].

In summary, there has been considerable interest in the polarization measurements of $B$-meson decays because they reveal both weak- and strong-interaction dynamics [34–36, 54]. New measurements will further elucidate the pattern of spin alignment measurements in rare $B$ decays, and further test the Standard Model and strong interaction dynamics, including the non-factorizable contributions to the $B$-decay amplitudes.

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