62. Form Factors for Semileptonic Kaon ($K_{\ell 3}$), Radiative Pion ($\pi_{\ell 2\gamma}$) and Kaon ($K_{\ell 2\gamma}$) Decays

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62.1 $\pi^{\pm}_{\ell 2\gamma}$ and $K^{\pm}_{\ell 2\gamma}$ Form Factors

The radiative decays, $\pi^{\pm} \to l^{\pm}\nu\gamma$ and $K^{\pm} \to l^{\pm}\nu\gamma$, with l standing for an e or a μ, and γ for a real or virtual photon (e^+e^- pair), provide a powerful tool to investigate the hadronic structure of pions and kaons. The structure-dependent part SD_i of the amplitude describes the emission of photons from virtual hadronic states, and is parametrized in terms of form factors V, A, (vector, axial vector), in the standard description [1–4]. Note that in the Listings and some literature, equivalent nomenclature F_V and F_A for the vector and axial form factors is often used. Exotic, non-standard contributions like $i = T, S$ (tensor, scalar) have also been considered. Apart from the SD terms, there is also the Inner Bremsstrahlung amplitude, IB, corresponding to photon radiation from external charged particles and described by Low theorem in terms of the physical decay $\pi^{\pm}(K^{\pm}) \to l^{\pm}\nu$. Experiments try to optimize their kinematics so as to minimize the IB part of the amplitude.

The SD amplitude in its standard form is given as

$$M(\mathrm{SD}_V) = \frac{-eG_F U_{qq'}}{\sqrt{2}m_P}\epsilon^{\mu} l^{\nu} V^P \epsilon_{\mu\nu\sigma\tau} k^{\sigma} q^{\tau} \tag{62.1}$$

$$M(\mathrm{SD}_A) = \frac{-ieG_F U_{qq'}}{\sqrt{2}m_P}\epsilon^{\mu} l^{\nu}\{A^P[(qk - k^2)g_{\mu\nu} - q_{\mu}k_{\nu}] + R^P k^2 g_{\mu\nu}\}\ , \tag{62.2}$$

which contains an additional axial form factor R^P which only can be accessed if the photon remains virtual. $U_{qq'}$ is the Cabibbo-Kobayashi-Maskawa mixing-matrix element; ϵ^{μ} is the polarization vector of the photon (or the effective vertex, $\epsilon^{\mu} = (e/k^2)\overline{u}(p_-)\gamma^{\mu}v(p_+)$, of the e^+e^- pair); $\ell^{\nu} = \overline{u}(p_{\nu})\gamma^{\nu}(1-\gamma_5)v(p_{\ell})$ is the lepton-neutrino current; q and k are the meson and photon four-momenta ($k = p_+ + p_-$ for virtual photons); and P stands for π or K.

For decay processes where the photon is real, the partial decay width can be written in analytical form as a sum of IB, SD, and IB/SD interference terms INT [1,4]:

$$\begin{aligned}\frac{d^2\Gamma_{P\to\ell\nu\gamma}}{dxdy} &= \frac{d^2\left(\Gamma_{\mathrm{IB}} + \Gamma_{\mathrm{SD}} + \Gamma_{\mathrm{INT}}\right)}{dxdy} \\ &= \frac{\alpha}{2\pi}\Gamma_{P\to\ell\nu}\frac{1}{(1-r)^2}\Bigg\{\mathrm{IB}(x,y) \\ &+ \frac{1}{r}\left(\frac{m_P}{2f_P}\right)^2\left[(V+A)^2\mathrm{SD}^+(x,y) + (V-A)^2\mathrm{SD}^-(x,y)\right] \\ &+ \epsilon_P\frac{m_P}{f_P}\left[(V+A)\mathrm{S}^+_{\mathrm{INT}}(x,y) + (V-A)\mathrm{S}^-_{\mathrm{INT}}(x,y)\right]\Bigg\}.\end{aligned} \tag{62.3}$$

Here

$$\begin{aligned}
\mathrm{IB}(x,y) &= \left[\frac{1-y+r}{x^2(x+y-1-r)}\right] \\
&\quad \left[x^2+2(1-x)(1-r)-\frac{2xr(1-r)}{x+y-1-r}\right] \\
\mathrm{SD}^+(x,y) &= (x+y-1-r)\Big[(x+y-1)(1-x)-r\Big] \\
\mathrm{SD}^-(x,y) &= (1-y+r)\Big[(1-x)(1-y)+r\Big] \\
\mathrm{S}^+_{\mathrm{INT}}(x,y) &= \left[\frac{1-y+r}{x(x+y-1-r)}\right]\Big[(1-x)(1-x-y)+r\Big] \\
\mathrm{S}^-_{\mathrm{INT}}(x,y) &= \left[\frac{1-y+r}{x(x+y-1-r)}\right]\Big[x^2-(1-x)(1-x-y)-r\Big]
\end{aligned} \tag{62.4}$$

where $x = 2E_\gamma/m_P$, $y = 2E_\ell/m_P$, $r = (m_\ell/m_P)^2$, f_P is the meson decay constant, and ϵ_P is +1 for pions and -1 for kaons. The structure dependent terms SD^+ and SD^- are shown in Fig. 1. The SD^- term is maximized in the same kinematic region where overwhelming IB term dominates (along $x+y=1$ diagonal). Thus experimental yields with less background are dominated by SD^+ contribution and proportional to $A^P + V^P$ making simultaneous precise determination of the form factors difficult.

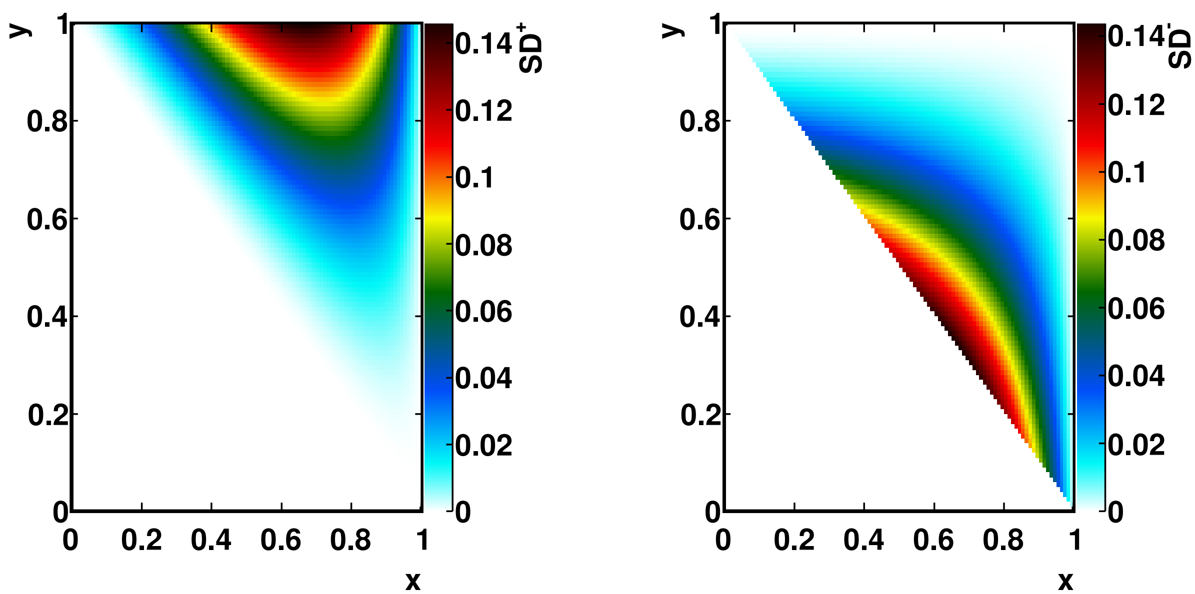

Figure 62.1: Components of the structure dependent terms of the decay width. Left: SD^+, right: SD^-

Recently, formulas 62.3 and 62.4 have been extended to describe polarized distributions in radiative meson and muon decays [5].

The "helicity" factor r is responsible for the enhancement of the SD over the IB amplitude in the decays $\pi^\pm \to e^\pm\nu\gamma$, while $\pi^\pm \to \mu^\pm\nu\gamma$ is dominated by IB. Interference terms are important for the decay $K^\pm \to \mu^\pm \nu\gamma$ [6], but contribute only a few percent correction to pion decays. However, they provide the basis for determining the signs of V and A. Radiative corrections to the decay $\pi^+ \to e^+\nu\gamma$ have to be taken into account in the analysis of the precision experiments. They make up to 4% corrections in the total decay rate [7]. In $\pi^\pm \to e^\pm \nu e^+ e^-$ and $K^\pm \to \ell^\pm \nu e^+ e^-$ decays, all three form factors, V^P, A^P, and R^P, can be determined [8, 9].

Theoretically, the first non-trivial χPT contributions to A^P and V^P appear at $\mathcal{O}(p^4)$ [4], respectively from Gasser-Leutwyler coefficients, L_i's, and the anomalous lagrangian:

$$A^P = \frac{4\sqrt{2}M_P}{F_\pi}(L_9^r + L_{10}^r), \quad V^P = \frac{\sqrt{2}M_P}{8\pi^2 F_\pi}. \tag{62.5}$$

In case of the kaon $A^K = 0.042$ and $V^K = 0.096$. $\mathcal{O}(p^6)$ contributions to A^K can be predicted accurately: they are flat in the momentum dependence and shift the $\mathcal{O}(p^4)$ value to 0.034. $\mathcal{O}(p^6)$ contributions to V^K are model dependent and can be approximated by a form factor linearly dependent on momentum. For example, when looking at the spread of results obtained within two different models, the constant piece of this linear form factor is shifted to 0.078 ± 0.005 [1, 2, 4].

We give the experimental $\pi^\pm$ form factors V^π, A^π, and R^π in the Listings. In the $K^\pm$ Listings, we give the extracted sum $A^K + V^K$ and difference $A^K - V^K$, as well as V^K, A^K and R^K. In particular KLOE has measured for the constant piece of the form factor $A^K + V^K = 0.125 \pm 0.007 \pm 0.001$ [10] while ISTRA+, $V^K - A^K = 0.21 \pm 0.04 \pm 0.04$ [11].

The pion vector form factor, V^π, is related via CVC (Conserved Vector Current) to the $\pi^0 \to \gamma\gamma$ decay width. The constant term is given by $|V^\pi(0)| = (1/\alpha)\sqrt{2\Gamma_{\pi^0\to\gamma\gamma}/\pi m_{\pi^0}}$ [3]. The resulting value, $V^\pi(0) = 0.0259(9)$, has been confirmed by calculations based on chiral perturbation theory (χPT) [4], and by two experiments given in the Listings.

A recent experiment by the PIBETA collaboration [12] obtained a $V^\pi(0)$ that is in excellent agreement with the CVC hypothesis. It also measured the slope parameter a in $V^\pi(s) = V^\pi(0)(1 + a \cdot s)$, where $s = (1 - 2E_\gamma/m_\pi)$, and E_γ is the gamma energy in the pion rest frame: $a = 0.095 \pm 0.058$. A functional dependence on s is expected for all form factors. It becomes non-negligible in the case of $V^\pi(s)$ when a wide range of photon momenta is recorded; proper treatment in the analysis of K decays is mandatory.

The form factor, R^P, can be related to the electromagnetic radius, r_P, of the meson [2]: $R^P = \frac{1}{3} m_P f_P \langle r_P^2 \rangle$ using PCAC (Partial Conserved Axial vector Current).

In lowest order χPT, the ratio A^π/V^π is related to the pion electric polarizability $\alpha_E = [\alpha/(8\pi^2 m_\pi f_\pi^2)] \times A^\pi/V^\pi$ [13]. Direct experimental and theoretical status of pion polarizability studies currently is not settled. Most recent theoretical predictions from χPT at $\mathcal{O}(p^6)$ [14] and experimental results from COMPASS collaboration [15] favor a small value of pion polarizability $\alpha_\pi \sim (2 \div 3) \times 10^{-4}\,\text{fm}^3$. Dispersive analysis of $\gamma\gamma \to \pi^+\pi^-$ crossection [16] and experimental results from MAMI collaboration [17] report a much larger value of $\alpha_\pi \sim 6 \times 10^{-4}\,\text{fm}^3$. Precise measurement of the pion form factors by PIBETA collaboration favors smaller values of polarizability $\alpha_\pi = 2.7^{+0.6}_{-0.5} \times 10^{-4}\,\text{fm}^3$.

Several searches for the exotic form factors F_T^π, F_T^K (tensor), and F_S^K (scalar) have been pursued in the past. In particular, F_T^π has been brought into focus by experimental as well as theoretical work [18]. New high-statistics data from the PIBETA collaboration have been re-analyzed together with an additional data set optimized for low backgrounds in the radiative pion decay. In particular, lower beam rates have been used in order to reduce the accidental background, thereby making the treatment of systematic uncertainties easier and more reliable. The PIBETA analysis now restricts F_T^π to the range $-5.2 \times 10^{-4} < F_T^\pi < 4.0 \times 10^{-4}$ at a 90% confidence limit [12]. This result is in excellent agreement with the most recent theoretical work [4].

Precision measurements of radiative pion and kaon decays are effective tools to study QCD in the non-perturbative region and are of interest beyond the scope of radiative decays. Meanwhile other processes such as $\pi^+ \to e^+\nu$ that seem to be better suited to search for new physics at the precision frontier are currently studied. The advantages of such process are the very accurate and reliable theoretical predictions and the more straightforward experimental analysis.

62.2 $K^{\pm}_{\ell 3}$ and $K^0_{\ell 3}$ Form Factors

Assuming that only the vector current contributes to $K \to \pi\ell\nu$ decays, we write the matrix element as

$$M \propto f_+(t)\left[(P_K + P_\pi)_\mu \bar{\ell}\gamma_\mu(1+\gamma_5)\nu\right] + f_-(t)\left[m_\ell \bar{\ell}(1+\gamma_5)\nu\right] \ , \quad (62.6)$$

where P_K and P_π are the four-momenta of the K and π mesons, m_ℓ is the lepton mass, and f_+ and f_- are dimensionless form factors which can depend only on $t = (P_K - P_\pi)^2$, the square of the four-momentum transfer to the leptons. If time-reversal invariance holds, f_+ and f_- are relatively real. $K_{\mu 3}$ experiments, discussed immediately below, measure f_+ and f_-, while K_{e3} experiments, discussed further below, are sensitive only to f_+ because the small electron mass makes the f_- term negligible.

62.2.1 $K_{\mu 3}$ *Decays*

Analyses of $K_{\mu 3}$ data frequently assume a linear dependence of f_+ and f_- on t, *i.e.*,

$$f_\pm(t) = f_\pm(0)\left[1 + \lambda_\pm(t/m^2_{\pi^+})\right] \ . \quad (62.7)$$

Most $K_{\mu 3}$ data are adequately described by formula 62.7 for f_+ and a constant f_- (*i.e.*, $\lambda_- = 0$).

There are two equivalent parametrizations commonly used in these analyses: $\lambda_+, \xi(0)$ parametrization and λ_+, λ_0 parametrization.

Older analyses of $K_{\mu 3}$ data often introduce the ratio of the two form factors

$$\xi(t) = f_-(t)/f_+(t) \ . \quad (62.8)$$

The $K_{\mu 3}$ decay distribution is then described by the two parameters λ_+ and $\xi(0)$ (assuming time reversal invariance and $\lambda_- = 0$).

More recent $K_{\mu 3}$ analyses have parametrized in terms of the form factors f_+ and f_0, which are associated with vector and scalar exchange, respectively, to the lepton pair. f_0 is related to f_+ and f_- by

$$f_0(t) = f_+(t) + \left[t/(m^2_K - m^2_\pi)\right] f_-(t) \ . \quad (62.9)$$

Here $f_0(0)$ must equal $f_+(0)$. The earlier assumption that f_+ is linear in t and f_- is constant leads to f_0 linear in t:

$$f_0(t) = f_0(0)\left[1 + \lambda_0(t/m^2_{\pi^+})\right] \ . \quad (62.10)$$

With the assumption that $f_0(0) = f_+(0)$, the two parametrizations, $(\lambda_+, \xi(0))$ and (λ_+, λ_0) are equivalent as long as correlation information is retained. (λ_+, λ_0) correlations tend to be less strong than $(\lambda_+, \xi(0))$ correlations.

Since the 2006 edition of the *Review* [19], we no longer quote results in the $(\lambda_+, \xi(0))$ parametrization. We have removed many older low statistics results from the Listings. See the 2004 version of this note [20] for these older results, and the 1982 version [21] for additional discussion of the $K^0_{\mu 3}$ parameters, correlations, and conversion between parametrizations.

More recent high-statistics experiments have included a quadratic term in the expansion of $f_+(t)$,

$$f_+(t) = f_+(0)\left[1 + \lambda'_+(t/m^2_{\pi^+}) + \frac{\lambda''_+}{2}(t/m^2_{\pi^+})^2\right] . \tag{62.11}$$

If there is a non-vanishing quadratic term, then λ_+ of formula 62.7 represents the average slope, which is then different from λ'_+. Our convention is to include the factor $\frac{1}{2}$ in the quadratic term, and to use m_{π^+} even for K^+_{e3} and $K^+_{\mu 3}$ decays. We have converted other's parametrizations to match our conventions, as noted in the beginning of the "$K^\pm_{\ell 3}$ and $K^0_{\ell 3}$ Form Factors" sections of the Listings.

There are two alternatives to the Taylor parametrization: The Pole Parametrization and Dispersive Parametrization.

The pole model describes the t-dependence of $f_+(t)$ and $f_0(t)$ in terms of the exchange of the lightest vector and scalar K^* mesons with masses M_V and M_S, respectively:

$$f_+(t) = f_+(0)\left[\frac{M_V^2}{M_V^2 - t}\right] , \quad f_0(t) = f_0(0)\left[\frac{M_S^2}{M_S^2 - t}\right] . \tag{62.12}$$

The Dispersive Parametrization approach, valid in a much wider kinematic range and able to describe at the same time τ-decay data, [22] uses dispersive techniques and the known low-energy K-π phases to parametrize the vector and scalar form factors:

$$f_+(t) = f_+(0)\exp\left[\frac{t}{m_\pi^2}(\Lambda_+ + H(t))\right]; \tag{62.13}$$

$$f_0(t) = f_+(0)\exp\left[\frac{t}{(m_K^2 - m_\pi^2)}\left(ln[C] - G(t)\right)\right], \tag{62.14}$$

where Λ_+ is the slope of the vector form factor, and $\ln C = \ln\left[f_0(m_K^2 - m_\pi^2)\right]$ is the logarithm of the scalar form factor at the Callan-Treiman point. The functions $H(t)$ and $G(t)$ are dispersive integrals.

62.2.2 K_{e3} *Decays*

Analysis of K_{e3} data is simpler than that of $K_{\mu 3}$ because the second term of the matrix element assuming a pure vector current [formula 62.6 above] can be neglected. Here f_+ can be assumed to be linear in t, in which case the linear coefficient λ_+ of formula 62.7 is determined, or quadratic, in which case the linear coefficient λ'_+ and quadratic coefficient λ''_+ of formula 62.11 are determined.

If we remove the assumption of a pure vector current, then the matrix element for the decay, in addition to the terms in formula 62.6, would contain

$$\begin{aligned} &+2m_K\, f_S\, \overline{\ell}(1+\gamma_5)\nu \\ &+(2f_T/m_K)(P_K)_\lambda(P_\pi)_\mu\, \overline{\ell}\,\sigma_{\lambda\mu}(1+\gamma_5)\nu \ , \end{aligned} \tag{62.15}$$

where f_S is the scalar form factor, and f_T is the tensor form factor. In the case of the K_{e3} decays where the f_- term can be neglected, experiments have yielded limits on $|f_S/f_+|$ and $|f_T/f_+|$.

For K_{e3} data, we determine best values for the three parametrizations: linear (λ_+), quadratic ($\lambda_+^{'}$, $\lambda_+^{''}$) and pole (M_V). For $K_{\mu 3}$ data, we determine best values for the three parametrizations: linear (λ_+, λ_0), quadratic ($\lambda_+^{'}, \lambda_+^{''}$, λ_0) and pole (M_V, M_S). We then assume $\mu - e$ universality so that we can combine K_{e3} and $K_{\mu 3}$ data, and again determine best values for the three parametrizations: linear (λ_+, λ_0), quadratic ($\lambda_+^{'}, \lambda_+^{''}$, λ_0), and pole (M_V, M_S). When there is more than one parameter, fits are done including input correlations. Simple averages suffice in the two K_{e3} cases where there is only one parameter: linear (λ_+) and pole (M_V).

A comprehensive global analysis of the semileptonic kaon decay data and its effect on the CKM unitarity debate can be found in [23,24]. An update on experimental data including NA48/2 newest results can be found in [25].

Both KTeV and KLOE see an improvement in the quality of their fits relative to linear fits when a quadratic term is introduced, as well as when the pole parametrization is used. The quadratic parametrization has the disadvantage that the quadratic parameter $\lambda_+^{''}$ is highly correlated with the linear parameter $\lambda_+^{'}$, in the neighborhood of 95%, and that neither parameter is very well determined. The pole fit has the same number of parameters as the linear fit, but yields slightly better fit probabilities, so that it would be advisable for all experiments to include the pole parametrization as one of their choices.

The "Kaon Particle Listings" show the results with and without assuming μ-e universality. The "Meson Summary Tables" show all of the results assuming μ-e universality, but most results not assuming μ-e universality are given only in the Listings.

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